AN ASSESSMENT OF GENETIC CONTROL OF FORM AND DIMENSION OF HUMAN DENTAL ARCHES

Harry L. Everett, D.D.S.

Christopher J. Matthews, D.M.D.

This paper submitted in partial fulfillment of the
requirements for a Certificate in Orthodontics
University of Oregon School of Dentistry

July 1978

ACKNOWLEDGMENTS

We would like to express our sincere thanks to the following people:

Dr. Ross Kaplan for his assistance, editorial guidance and relentless coordination of all interdisciplinary aspects involved in this investigation.

Dr. A.C. Brown for assistance in programming, digitizing and transmitting the raw data for analysis.

Dr. K. Lu and Bill Koshow for statistical and computer handling of the data.

Dr. Paula Kanarek for statistical interpretation and editorial aid.

Dr. Bhim Savara and staff for the use of records from the Child Study Clinic, University of Oregon Dental School.

Dr. Peter Shapiro for the generous use of records from the Department of Orthodontics, University of Washington.

Ethel Hauser and Barbara Holcomb for their aid in processing the photographic records.

Liz Donovan for the typing and preparation of this manuscript.

TABLE OF CONTENTS

INTRODUCTION

The dental arch form of man, though characteristically human, presents enough variation that dentists and anthropologists have long been accustomed to categorizing the distinctive shapes found in human dental arches. This variation has been described architecturally in terms such as gothic and romanesque, geometrically as parabolas, ellipses, and functions of catenary curves, or, most recently, translated into the more flexible mathematical form of polynomial equations. Yet the clinician still prefers the most practical and easily understood of classifications. He sees dental arch forms as rounded, or square, or ovoid, or tapering, or some combination thereof. What these terms lack in objectivity they compensate for in utility, and the particular use to which the clinical orthodontist puts them bears a distinct implication.

When the orthodontist speaks of a tapering arch, he is describing normal variation in dental arch form, but in terms of treatment objectives he is implying more than simple description. Most orthodontists prefer to incorporate in their treatment philosophies a desire to preserve in some part the arch form found in the dental arches of their patients prior to treatment, and motives for doing so generally reflect an attempt to enhance stability, function, and aesthetics in the final treatment results. Sanctity of the original arch form, however, is a concept based heavily on clinical experience; no hard evidence exists to show that the variation found in the <u>shape</u>

of dental arches should influence an orthodontist's treatment goals.

Genetic studies seldom verify such empirical concepts, but by clarifying the processes through which normal variation occurs, they do provide valuable background data for more specific clinical studies. This study, then, will employ the twin method in an attempt to unravel the roles that genetics and environment play in determining dental arch form. Within-pair variance in arch form will be statistically compared among samples of monozygotic twins, like-sexed dizygotic twins, and a control group of like-sexed siblings. In addition, within-pair variance in intercanine and intermolar widths and arch circumference will be subjected to the same statistical comparison. Because it appears certain that arch form manifests itself as a continuous variable, the arch forms in these samples will be translated into quantitative characters by the use of polynomial equations. Due to the limitations of this type of study, an attempt will not be made to assign relative degrees of influence to heredity and environment. Rather, our purpose will be only to determine whether genetics or environment, or both, play a statistically significant role in determining the form and dimensions of an individual's dental arches.

REVIEW OF THE LITERATURE

Even the very early dental literature demonstrated an interest in the effect that heredity has upon the features of the oral cavity. In the 1870 Dental Cosmos M'Quillen[1] conjectured about "the transmission of hereditary peculiarities in the dental organs" in light of the recent revelations of Darwin in The Origin of Species. Typical of the era which followed were the pedigree studies which attempted to assign simple Mendelian inheritance patterns to dental anomalies[2,3,4] in 1935 or facial deformities such as cleft lip[5].

Keeler[4] in 1935 regretted that insufficient research had been taking place in dental genetics but was apparently unaware of studies already published which were more sophisticated than his own pedigree searches. Bachrach and Young[6] in 1927 and Goldberg[7] in 1929 had published innovative twin studies of dental occlusion, and Korkhaus[8] in 1930 had followed with an investigation which explored pathological as well as normal variation in fraternal and identical twins.

Iwagaki[9] in 1938 also broke away from the pedigree model with a population study of the effects of heredity on progenia, a condition which includes both mandibular protrusion and edge-to-edge bite. Although unsuccessful in his attempt to discern Mendelian inheritance by using statistical data, he concluded that progenia was familial and suggested that the mode of inheritance may be Mendelian recessive. Rubbrecht[10] returned to the pedigree study in 1939 and concluded that mandibular prognathism had been proven to be dependent on heredity by its frequency and distribution over a large number of families, and that its mode of inheritance is "irregularly dominant." Johnson[11] followed in 1940 with a study of dental occlusion and skull form in dogs, concluding that genetic influences condition the environmental effects on facial growth.

The conclusions drawn by Iwagaki, Rubbrecht, and Johnson, however, were not entirely justified by the studies from which they were derived, and this tendency toward speculation was carried even further by Hughes and Moore[12] in the early forties. Although they were among the first to support the concept of multiple gene inheritance in the craniofacial complex, their rendering of hereditary and environmental control into set percentages is of questionable validity. Drawing upon a sample of 554 individuals from 150 families, they determined that heredity plays a strong role in controlling craniofacial morphology and growth, and then broke down the causes of specific malocclusions into percentages attributable to heredity, to environment, and to interaction between the two. Hughes[13] was also a strong advocate of the concept of genetic independence of anatomic parts, even to the point of attributing single gene inheritance to a trait such as palatal height.

Wylie[14] in 1944 took a more objective approach by quantifying craniofacial features through the use of cephalometric measurements. His sample consisted of fifteen families, thirteen of which included like-sexed twins, and although none of the linear and angular measurements showed significant genetic variation, his technique of using cephalograms to measure craniofacial variables was the basis of many later studies. Wylie did not determine zygosity on his twin sample but suggested that this method would prove valuable in twin studies where zygosity had been ascertained.

Two years earlier Cohen[15] had conducted a twin study of dental arch form, but his objective was to use arch shape as a means of determining zygosity. Although his investigation proved valuable primarily as a study of dental arch form, zygosity determination did become more precise in the ensuing years, especially through the use of blood group factors. And, as Wylie had suggested, many researchers followed with cephalometric studies that included twins whose zygosity had been determined to a high degree of probability.

Lundstrom[16,17] reported a cephalometric twin study in 1954 and 1955 in which he had used a sample of 50 pairs of monozygotic twins and 50 pairs of dizygotic twins. The cephalometric distances and angles measured were similar to Wylie's, and Lundstrom concluded that genetic factors exert a greater influence over these measurements than non-genetic factors. In an earlier investigation Lundstrom[18] had analyzed tooth size and dental occlusion in a sample of 100 monozygotic and 102 dizygotic twins, probably the first such study to consider all characteristics as quantitative continuous variables. Walker[19], in a review of the earlier study, was critical of Lundstrom's failure to utilize blood groups or dermatoglyphics for zygosity determination, but Lundstrom countered in the later cephalometric studies by reporting that blood groupings had been done on fourteen of the twin pairs used, none of which contradicted the previous zygosity assignations.

Horowitz, Osborne, and DeGeorge[20] were more rigorous in diagnosing zygosity for their cephalometric twin study of 1960. Using cephalometric tracings of thirty-five monozygotic and twenty-one like-sexed dizygotic twin pairs, they compared linear measurements and found "highly significant variation" occurring in anterior cranial base, mandibular body length, total face height, and lower face height.

Hunter[21] in 1965 conducted a cephalometric study in which he compared linear measurements in seventy-two sets of twins, thirty-seven monozygotic and thirty-five dizygotic. His purpose was to evaluate -- in cephalometric terms -- the conclusion by Osborne and DeGeorge[22] in 1959 that generally measurements taken along the long axis of the body show the strongest genetic component. Hunter agreed, finding that on the whole, dimensions of facial skeletal height showed a "significantly higher component of genetic variability" than those of facial skeletal depth.

Snodgrasse[23] in 1948 and Stein, Kelly, and Wood[24] in 1956 also conducted studies using cephalometric measurements to quantify craniofacial variation. Snodgrasse's family line study included one set of twins, and he concluded that familial records aid the clinical orthodontist in anticipating difficulty in treatment as well as prognosis for success. Stein et al[24] studied a large group of female college students, their parents, and their siblings, the sample including four sets of twins; only angular measurements were used. The resultant correlation coefficients indicated a greater genetic significance between siblings than between parent-sibling combinations.

Noyes, in a 1958 review of the question of genetic influence on malocclusion, noted the "need for establishing genetic units within the face rather than seeking genetic significance in orthodontic diagnostic criteria."[25] Curtner[26] in a 1953 study including three sets of twins had expressed a similar view in suggesting the value of superimposing headfilms in studies of human inheritance rather than measuring lines and angles. The most prominent study espousing the deficiencies in using line and angle constructs for cephalometric studies of heredity was that of Kraus, Wise, and Frei[27] in 1959. They used six sets of triplets and found superimposed bony profiles, scored visually and subjectively for concordance or discordance, to be of more value in studying genetic control of craniofacial morphology than measuring lines and angles. Moorrees[28] also used a superimposition technique to assess familial patterns in facial proportions except that he divided his cephalometric tracings into a series of vertical and horizontal planes and superimposed these rather than profiles.

However, Goodman[29] noted in 1965 the limitations of superimposition techniques, such as that proposed by Kraus et al, due to the subjectivity of these methods. He called for a more quantifiable means of assessing genetic control in craniofacial traits; while earlier studies did utilize quantitative measures, they were measuring artificial constructs that had no coherent genetic validity.

Thus Watnick[30], in a 1972 cephalometric study of seventy pairs of twins, half monozygotic and half dizygotic, attempted to quantify small, definable anatomic units in the mandible which represented local growth sites. Then he statistically assessed the predominance of genetic or environmental influences upon the morphology of these anatomic subsets. Watnik contended that "it is not the number of measurements or variables that determine biologic relevance, but the nature of the measurement itself," and that the advantage of his method lay in "the quantification of morphologic units and their relatively small size."

Nevertheless, the criterion of "biologic relevance" would apparently be met by cephalometric measurements which are non-morphologic in character, as long as they are expressions of anatomic realities. For example, the traditional twin study published by Arya et al[31] in 1973 sought to assess heritability of linear cephalometric dimensions in the mandible. Although strictly speaking these dimensions are artificial constructs, they do reflect biologically relevant attributes of the mandible such as length, width, and height. In similar fashion the heritability studies of palatal dimensions by Shapiro in 1969[32] and Riquelme and Green[33] in 1970 dealt with measurements which are man-made concepts rather than strict morphologic units. Yet they are relevant in the sense that they express quantitative attributes of the anatomic unit known as the palate.

Dental arch form is probably an artificial construct as well, at least when conceptualized as it usually is, in the form of a two-dimensional curve. But again, this man-made concept is an expression of anatomic realities; it is a means of describing one particularly distinctive quality of the dental arch -- its shape. The shape of a dental arch, however, is obviously more difficult to quantify than linear or angular dimensions. And quantifying a complex, often asymmetrical, curve is not the only obstacle encountered in a heritability study of dental arch form. The twin method itself presents its own inherent difficulties.

The complex nature of multifactorial inheritance has long been a major difficulty in studying the influence of heredity upon the morphology of the craniofacial complex, and in particular the dental arch. It is generally accepted that the characteristics of normal occlusal variation are polygenic, or multifactorial, and are therefore represented by continuous phenotypic variation[34,35,36,37,38,39,40,41]. Although extreme deviations of occlusion associated with craniofacial anomalies are found to have single gene inheritance patterns[39], the general inheritance patterns of occlusal morphology are thought to be polygenic.

Polygenic inheritance accounts for a wide variation in the phenotype whereas the single locus concept, even considering multiple alleles, cannot account for the broad range of variation found in the population. The theory of polygenic inheritance is based on the concept that phenotypic expression of a characteristic is determined by the summative effects of a large number of genes and the overlying environmental influence. The genes collaborate in their action, and the varieties within a population can be expressed in graded phenotypes which tend to be continuous and normally distributed[41].

One approach for studying these traits is the method of comparing resemblance between relatives. Fisher[42] has shown that the theoretical correlations between relatives for a trait should be proportional to the number of genes in common, provided that: the trait is determined purely by heredity; the genes concerned are strictly additive in their effects; and mating is random for the trait under investigation. Under these circumstances the correlation coefficient is utilized as a quantitative measure of resemblance between relatives.

The identical-fraternal twin model has been widely used to partition off the relative influence genetics and environment have upon a phenotype. This method, according to Osborne and DeGeorge, "constitutes the most efficient approach for appraising the heredity-environment problem in man, particularly with respect to complex or multifactorial inheritance."[22] It is assumed in the twin method that monozygotic twins are derived from a single fertilized ovum and thus have identical genetic endowments and a coefficient of genetic relationship of 1.0. Osborne and DeGeorge go even further, stating that since monozygotic twins are identical for their whole genetic constitution, they are identical not only for all genetic factors that have a modifying influence on the penetrance, or expressivity, of a major gene, but also in the degree of their response to specific environmental influences. Thus they have identical genetic buffering systems which, by homeostatic modifications of developmental or psychological patterns, will protect equally both members of the pair through the usual range of environmental variations[22].

Dizygotic twins, on the other hand, are derived from two separate fertilized ova and have the same genetic similarity as full siblings, the average coefficient of genetic relationships being .5. Any difference in monozygotic twins is assumed to be due to environmental influence since their genetic makeup is identical; differences in the phenotypes of dizygotic twins results from both genetic and environmental influences. This is a basic assumption of the twin study.

Obviously the reliability of zygosity determination is of fundamental importance in twin studies. Zygosity determination in the early studies was based on some or all of the following characteristics: visual impression of resemblance; hair and eye color; hair whorl; configuration of homologous ears; anthropologic measurements; similarity of fingerprints; and palm and sole patterns. All of these determinants employ a certain degree of subjectivity in diagnosis, and recent improvements in zygosity determination have resulted from the use of blood group studies and dermatoglyphic ridge counts, both of which utilize a quantitative approach to the problem. Of the two, diagnosis by blood groups is probably the more reliable method because of the precise phenotype-genotype relationship involved. An example of modern day zygosity determination is reported in the 1973 cephalometric twin study by Arya et al[31], in which the probability of dizygosity for concordant twins was established at less than 5%.

Questions of zygosity need no longer compromise twin study findings, but biases which are inherent in the method itself must be considered. These biases may be categorized as statistical and biologic. Of the biologic biases, Price[43] has outlined three which may affect the similarity of twins: position and crowding in utero; time of scission of monozygotic twins; and mutual vascular circulation in utero. Newman et al[44] believed that differences in blood supply due to unequal blood exchanges between foetuses sharing a common placenta may cause monozygotic twins to differ more in size than dizygotic ones. Some authors[45,46] suggest also the possibility of a third type of twinning in which the ovum divides prior to fertilization, each ovum being fertilized by a different sperm.

Statistical biases result from failure to meet the assumptions made for the statistical tests used. The most common of these assumptions are that: random mating exists in the population under study; no linkage exists for craniofacial growth loci; no differential variability or fecundity exists in the population; and the magnitude of environmental differences between monozygotic and dizygotic twins is the same. All these assumptions can be disputed, thus clouding the interpretations of co-variability of relatives in human populations.

Bearing in mind all the problems associated with the twin method, it still remains the best means of evaluating the relative roles of genetic and environmental factors in the development of complex traits. Allan in 1965 noted that "despite the many difficulties of twin research ... no material but twins can provide such convincing evidence for environmental etiologic factors prior to the demonstration of the factors individually."[47]

Several early authors used the twin method in their investigation of the relative importance of heredity and environment in the development of the dental arches. Bachrach and Young[6] found in 1927 that concordance of normal occlusion is significantly higher in identical pairs than in fraternal twins, and that identical twins present a closer coincidence in this type of malocclusion than do fraternal twins of like or unlike sex. Goldberg[7] in 1929 concluded that tooth position is not determined by physical forces. Hereditary factors determine arch form, and environment, whether intraoral or extraoral, under average conditions plays a small role in the determination of arch shape. And Korkhaus[8] in 1930 recognized differences in identical twins due to extrinsic factors but maintained that for the most part his identical sets were strikingly similar in their dental conditions.

Newton[48] reported in 1937 on a set of triplets in which two members were found to have similar dental arch forms while a third was dissimilar. Braun[49] found in 1938 that dental arches of fraternal twins were more variable than that of identical twins but more alike than those of unrelated pairs selected at random from the twin sets. Thus he concluded that heredity plays the dominant role in determining the size and shape of the dental arches.

In 1942 Cohen, Oliver, and Bernick[50] reported an investigation into the use of dental arches as a possible adjunct to the current method of zygosity determination. Dental arches of nine sets of triplets were compared and zygosity determination was made based on dental arch measurements. The results were compared with that of conventional zygosity tests. They concluded that dental arch measurements were more variable than other criteria used for zygosity testing and therefore not a good method for determination of monovular origin. Cohen et al[15] followed with a study of arch patterns in the same set of triplets and found that, in general, the arch forms of identical pairs were more alike than the arch forms in non-identical pairs; however, considerable variation was observed in three of the five identical pairs from the triplet sample. They concluded that the use of arch pattern is more reliable than arch dimension in the determination of monovular origin.

Although zygosity determination has become much more accurate since the time of Cohen's work, the literature shows a decided absence of quantitative twin studies investigating dental arch form and dimension. One exception is the study published by Menezes, Foster, and Lavelle[51] in 1974 in which three sets of triplets were used to appraise genetic control in the dimensions of the teeth and dental arches. Two of the triplet sets were entirely monozygotic; the third set consisted of only two monozygotic members thus yielding two dizygotic pairs. Arch width and length were measured and statistically compared, as were tooth dimensions. As expected, tooth dimensions were found to be under considerable genetic control, but, they concluded, dental arch width and length are not. A substantial limitation of this study, however, was the small sample size.

It is not uncommon to find studies published in which the dental arches of one set of twins are subjectively compared, especially those that show marked contrasts between supposedly monovular twins. Two recent investigations which qualitatively compare arch form in one set of monozygotic twins are those of Sakuda[52] in 1973 and Becker[53] in 1977. In both instances zygosity was established through blood groupings and dermatoglyphics. Sakuda found that the shapes of the dental arches in his sample were similar, with some environmental differences. Becker concluded that the minor differences in the arch forms of his set of twins could be attributed to a thumb sucking habit in one, as well as to individual variation within the same genetic pattern.

Aside from limited sample sizes, one of the major obstacles in assessing the influence of heredity and environment upon arch form has been the inability or reluctance of investigators to quantitatively define dental arch shape in a flexible mathematic form which lends itself to statistical analysis.

Dental arch shape has been described qualitatively in such varied terms as semiellipsoid, U-shaped, rotund, paraboloid, and horseshoe-shaped. The ellipse and parabola in particular have been used historically to describe the morphology of the dental arch. Black in his text on dental anatomy stated that "the upper teeth are arranged in a semi-ellipse ... (while) the lower teeth are arranged similarly, on a smaller curve."[54] Angle asserted in his discussion of the line of occlusion that "the form of the line (of occlusion) resembles a paraboloid curve and varies within the limits of normal according to the race, type and temperament of the individual."[55] Sicher[56] noted that the shape of the dental arch varies considerably, but in the average individual the upper arch can be described as elliptical, the lower arch as parabolic.

Many investigators have studied the dental arch, and numerous opinions have emerged regarding its shape. The catenary curve, originally described by Galileo in the 16th century[57], has been proposed by several authors to most accurately describe the "common line of occlusion." McConaill and Scher in an attempt at a physiologic justification of the catenary curve pointed out in 1949 that "the catenary, like a straight line, is a curve of minimal extraneous force"[58] and therefore is the simplest form in which the teeth can be arranged. Scott[59] concurred and has stated that not only do the permanent upper and lower dental arches conform to the catenary curve, but also that the arrangement of the embryonic tooth germs lies in the shape of a catenary as well. Burdi[60] investigated this claim in 1968 and found it to be true of embryos over the 8½ week stage of embryogenesis.

Currier[61] in 1969 fit parabolic and elliptical curves to landmarks on radiographs of dental casts from a sample of Caucasians with ideal occlusions. Using selected points on the teeth converted to x-y coordinates, a least squares curve fitting program was used by a computer to select an ellipse and parabola which most closely approximated each dental arch. Statistical analysis concluded that the ellipse provided a better "goodness of fit" for the maxillary arch while the parabola provided a better fit for the mandibular arch.

Brader[62] in 1972 proposed a three focal ellipse with the teeth occupying the position at the constricted end. This construction derived from a physiologic force theory in which rest tissue forces are the primary determinants of arch form.

In 1975 Biggerstaff[63] reported fitting a generalized quadratic equation, i.e., a set of equations including an ellipse, a parabola, and a hyperbola, to seven individuals with good occlusion. The casts were oriented and, after identifying anatomic landmarks with inked points, were photographed. The points of interest were then converted to x-y coordinates and analyzed by computer. The relative goodness of fit of the line of occlusion was studied, but no conclusions were drawn.

Lu[64] had already proposed in 1966 that for practical purposes the representation of general arch form seldom requires more than a fourth degree orthogonal polynomial equation of the form $y=a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$. The even powered polynomials are by definition symmetric and thus are a measure of the symmetry of the arch while the odd powered polynomials are by definition asymmetric and therefore measure the asymmetry.

Although curves were not fit to actual data points from dental arches in this study, Lu maintained that the fourth degree polynomial is a mathematic form with sufficient flexibility to accurately describe two dimensional representations of dental arch form.

Pepe[65] in 1975 published a study in which curves generated by even powered standard polynomials, second through eighth degree, and by catenary equations as well, were fit to data representing the anatomic landmarks constituting the line of common occlusion. The material for this study consisted of the same seven individuals with good occlusion as used in the Biggerstaff investigations. Anatomic landmarks on the casts were photographed, digitized, and, using the least squares solution, the mathematic formulae were fit to the raw data. The mean square error was computed for all polynomials and catenaries which had been fit to the dental arches. Pepe concluded that the catenary equation is less descriptive of dental arch form than the sixth degree polynomial equation which afforded significant increase in accuracy of fit over a fourth degree polynomial equation. She did not however statistically test that the sixth degree polynomial was significantly different from the fourth degree equation.

MATERIALS AND METHODS

Materials

The sample for this study was composed of 22 pairs of monozygotic twins, 11 male and 11 female; 13 pairs of like-sexed dizygotic twins, 7 male and 6 female; and 21 pairs of like-sexed siblings*, 8 male and 13 female. All subjects were Caucasian, predominantly of northwest European ancestry and of middle socioeconomic status.

The records of the bulk of the sample were collected as part of the longitudinal growth study conducted by the Child Study Clinic at the University of Oregon Health Sciences Center. These individuals resided in the greater metropolitan area of Portland, Oregon. Cases were selected for this study on the basis of complete zygosity diagnostic results and the availability of study casts of the intact permanent dentition.

Five pairs of dizygotic "twins" were obtained from records at the Department of Orthodontics, University of Washington Health Sciences Center. These cases were originally included in a study conducted by Kraus, Wise and Frei[27], and each pair was part of a triplet set. These cases were selected at random from the available triplet sets having dizygotic relationships.

Dental casts of the permanent dentition were obtained for the entire sample. Impressions had been taken in alginate and promptly poured in orthodontic plaster. The ages at which the casts were

* Siblings in this sample are exclusive of twins.

obtained were the same for each member in each pair except in 16 pairs where the age difference ranged from 1 to 4 years, but in all these cases the individuals were in the late teens or early twenties. The ages of the pairs ranged in the monozygotic group from 12 to 24 years with a mean of 16.27 years; in the dizygotic group from 10 to 19 years with a mean of 13.87 years, and the mean age for the siblings was 17.69 years with a range of 10 to 24 years. None of the individuals included in the study had undergone orthodontic therapy prior to the time at which casts were obtained.

The zygosity of the University of Oregon sample was determined on the basis of blood group systems. Blood samples were collected from the subjects and both parents, and the serological workup was carried out at the University of Oregon Medical School. Blood group systems tested were: ABO, MNS, Rh, P, Kell, Duffy, and Kidd. The serum antibodies tested were: A, Ai, B; M, N, S; C, D, E, c, e, C^w; P; K; K, k, Kp^b; Fy^a; jk^a, jk^b. Discordance for any one of these antisera was regarded as sufficient evidence for dizygosity. In addition to the blood groupings, the diagnosis of zygosity was supplemented by dermatoglyphics, phenylthiocarbamide taste testing and concordance of physical characteristics.

The zygosity diagnosis in the University of Washington sample was based on blood group data, dermatoglyphics, phenylthiocarbamide taste testing, dental morphology, photographs, and physical characteristics as described by Kraus, et al[27].

Methods

After the casts were oriented on surveyor tables, dental and soft tissue landmarks were identified with ink and photographed in the following manner:

The lower cast was placed in a surveyor table which was adjusted so that the occlusal plane of the cast was parallel to the base of the surveyor table. The occlusal plane was defined as the plane formed by the distobuccal cusp tip of both lower first molars and the buccal cusp tip of the lower right first bicuspid, modified after the method used by Moyers, Van der Linden, Riolo and McNamara[66].

The upper cast was then placed in occlusion on the lower cast held in the surveyor table. Three horizontal cast orientation marks were scribed on the base of the upper cast, two on either side of the heel and one on the front of the base (see Figure 1), all three scribe marks being the same distance from the base of the surveyor table. Using the two heel scribe marks as a guide for a protractor, two vertical orientation marks were scribed on the heel of both upper and lower casts (see Figure 1). Then these marks were projected onto the occlusal surface of each cast (Figure 2, #1).

The upper cast was then mounted on a surveyor table and the three horizontal cast marks were oriented equidistant from the base of the surveyor table.

With both casts on surveyor tables and the defined occlusal plane parallel to the surveyor table bases, the anatomic landmarks were marked with water soluble ink (Sanford's Vis-a-vis, black). The soft tissue landmarks used in this study were: (a) the most dorsal indication of the midpalatal raphe (Figure 2, #2); (b) the most ventral point on the midpalatal raphe (Figure 2, #3); and (c) the lateral termination of the most anterior pair of rugae (Figure 2, #4). The dental landmarks utilized were: (a) the buccal cusp tips of the molars and premolars;

(b) the cusp tip of the canines; and (c) the midpoint on the incisal edge of the incisors.

The casts were photographed with a 35 mm Nikkormat single lens reflex camera with a 100 mm lens and bellows using Kodak Panatomic X black and white film (ASA 32). To insure a fixed focal distance on all casts photographed and to facilitate standardized enlargement, an orientation table was constructed as shown in Figure 3. The camera was mounted on a tripod and kept at a constant distance above the orientation table. The casts mounted on surveyor tables were then raised on a laboratory jack through an aperture in the orientation table so that the occlusal plane was level with the surface of the orientation table.

Standardized enlargement of the negatives was controlled through the use of a millimeter ruled graph paper scale on the surface of the orientation table. This millimeter scale was employed as a guide for the enlargement of all prints to 2½ times actual size to facilitate the digitizing procedure and minimize errors.

Four fiducial marks were marked on the graph paper on the surface of the orientation table to form a rectangle 80 mm apart on the horizontal and 50 mm apart on the vertical axes (see Figure 4). These fiducial marks were utilized for subsequent computer correction of all measurements to actual size.

The enlarged photographic prints were taped to the surface of a graphic digitizing board (Summagraphics Data Tablet Digitizer HW-1-20), which records the x and y coordinates of each point to the nearest tenth of a millimeter. Fiducial points, heel marks, and anatomic landmarks were digitized as x, y coordinates and relayed to a Digital

Equipment Corp. PDP 11/45 computer programmed to store the raw data on a disc cartridge.

The data was then transferred via a modem attached to telephone lines to a CDC computer located at Oregon State University for scaling, reorientation, and statistical analysis.

Scaling and Transformation

A program was written to convert the x, y coordinates from the photographs which had been enlarged 2½ times, back to the original scale.

The scaling procedure was computed in the following manner: Utilizing the known distances between the fiducial marks on each photograph (80 mm on the horizontal, 50 mm on the vertical axis) and the average unscaled distances as derived from the x,y coordinates, scaling factors were computed as a ratio of the original distance to the digitized distance. Each coordinate was then multiplied by the appropriate scaling factor to obtain scaled data.

Another program was written to perform a series of transformations and rotations on the coordinates of the scaled points. First the lower arch points were flipped over to obtain the normal occlusal relationship between arches and oriented relative to the upper arch through superimposition of the digitized heel marks. When superimposition was not exact, the heel marks were aligned along a straight line with both upper and lower right and left heel marks equidistant from each other.

Then the digitized points of the two arches were rotated and transformed so that the midline of the upper arch, as determined by the midpalatal raphe marks, was located along the y axis. The upper and lower arches were then moved along the y axis so that the most anterior point of each arch lay on the x axis.

Statistical Analysis

Following scaling and transformation, standard polynomial equations, first through fourth degree, were fit by the method of least squares in a step-wise fashion to both upper and lower arches of each individual:

$$\hat{y} = B_{o_a} + B_{1_a} x^1$$

$$\hat{y} = B_{o_b} + B_{1_b} x^1 + B_{2_b} x^2$$

$$\hat{y} = B_{o_c} + B_{1_c} x^1 + B_{2_c} x^2 + B_{3_c} x^3$$

$$\hat{y} = B_{o_d} + B_{1_d} x^1 + B_{2_d} x^2 + B_{3_d} x^3 + B_{4_d} x^4$$

The method of least squares derives the coefficients that minimize the sum of squares:

$$\sum_{i}^{n} (y_i - \hat{y}_i)^2$$

where y_i is the y coordinate of the data point and $\hat{y}_i$ is the value predicted by the polynomial. At each succeeding polynomial the coefficient of the highest power of x, B_{1_a}, B_{2_b}, B_{3_c}, B_{4_d} was selected for each arch for statistical analysis. Coefficients derived in this manner were assumed to be independent of each other, continuous, and normally distributed. The within-pair mean square error of these coefficients was computed and compared among the three groups by F tests. All ratios were tested at the 0.05 level of significance. At each step the coefficient of determination for that polynomial equation was determined as well as the F ratios (see Appendices 1-3).

The coefficients are expressed in the following units: tenths of a millimeter y per tenths of a millimeter x, raised to the appropriate power. The mean square error is expressed in the above units squared.

In addition, the following variables were derived from the digitized data for each arch: intercanine width, intermolar width, and arch circumference. Intercanine width was measured from canine cusp tip to cusp tip while intermolar width was measured from the mesiobuccal cusp tip of one first molar to the mesiobuccal cusp tip of the contralateral first molar. In the event that one of the data points needed to compute intercanine or intermolar widths was missing, that dimension was not computed for that set of twins. Arch circumference was computed by sequentially summing the distance between each data point beginning with the distobuccal point of one first molar around to the distobuccal point of the opposite molar. The within-pair mean square error of each variable was computed and compared among the three groups using F ratios.

For all variables, the following research hypotheses were evaluated:

1 - the within-pair variance for monozygotic twins is less than for dizygotic twins;

2 - the within-pair variance for monozygotic twins is less than for siblings;

3 - the within-pair variance for dizygotic twins is less than for siblings.

<u>Error of the Method</u>

The upper and lower casts of twelve subjects were randomly selected by means of a random number table and the entire procedure - marking of casts, cast orientation, marking anatomic points, photography, enlargement, and digitizing - was repeated. For each replicated case the first through fourth degree coefficients were derived and compared to the original values. In addition, the measured variables - intercanine width, intermolar width, and arch circumference - were computed and compared to the original values. The standard error of the method was derived for each coefficient and each measured variable:

$$S.E.M. = \sqrt{\frac{\Sigma(d)^2}{2N}}$$

where d is the difference between the two measurements.

FINDINGS

The coefficient of determination (R^2) obtained with the fourth degree polynomial equation will be observed from Table 1 to account for a mean of 0.979 of the variation in the lower arch and 0.982 in the upper arch. The range was from .89 to 1.00 in both arches indicating not only that the fourth degree polynomials have the flexibility to adequately describe the dental arches on the average but also the range was quite small.

By observing the R^2 values in the table it is apparent that the first degree polynomial equation contributed very little to the overall fit of the polynomial to the dental arch. The second degree polynomial makes the major contribution averaging .957 in the upper and .962 in the lower. The third degree coefficient adds minimally to the fit of the second degree polynomial. The fourth degree polynomial explains approximately 43% of the remaining unexplained variation from the third degree term. In summary, these results show that the fourth degree polynomial accounts for an average of 98% of the variation in the upper and lower arches. The vast majority of this variation was accounted for by the contribution of the second order term with the fourth degree term making a minor contribution.

In Table 2A, the within-pair mean square errors of the coefficients of the polynomials are tabulated. By examining the mean square errors obtained from the four coefficients in the upper arch there appears to be no striking difference between the three groups. Table 2B reveals that in fact none of the F ratios obtained by comparing the mean square errors of any two groups were statistically significant. In summary, there appears to be no significant differences among the three groups for each of the four coefficients in the upper arch.

In Table 3A the within-pair mean square error of the coefficients for the lower arch is tabulated. There appears to be less intrapair variance in the monozygotic group than in the dizygotic or sibling groups for the B_{2_b} and B_{4_d} coefficients. The results of the F tests in Table 3B indicate that there is significantly less intrapair variation in the monozygotic group in the second degree coefficient than in the dizygotic or sibling groups. The intrapair variance of the fourth order coefficient in the monozygotic group was significantly less than in the dizygotic group. However the difference in variance between the monozygotic and sibling groups was not significant. Where the sibling and dizygotic groups were compared, no significant differences were observed.

Table 4 illustrates that in the upper arch the within-pair variance of intercanine width and intermolar width of the three groups show no significant differences. In the sibling/monozygotic comparison for intermolar width, the F ratio (2.07) was borderline. However, in the arch circumference of the upper arch there was significantly less within-pair variation among monozygotic twins than among dizygotic and sibling pairs.

In the lower arch, comparison of intrapair variance for the monozygotic group was significantly less than for either dizygotic or sibling groups for intercanine width, intermolar width and arch circumference. No significant F ratios were observed between the siblings and dizygotic pairs.

Error of the Method

The standard errors of the method for the coefficients were:

S.E.M. $B_{1_a} = 1.3743 \times 10^{-2}$

S.E.M. $B_{2_b} = 0.0625 \times 10^{-3}$

S.E.M. $B_{3_c} = 1.7435 \times 10^{-6}$

S.E.M. $B_{4_d} = 0.4028 \times 10^{-8}$

The standard error of the method was very small for B_{2_b} relative to the within-pair standard error ($\sqrt{MSE}$) for that coefficient in each arch and in each group (Tables 2 and 3). The standard error of the method for B_{4_d} was also small.

More variability in the reproducibility of the coefficients B_{1_a} and B_{3_c} was evident. But as the contribution of these coefficients to the polynomial equations was minimal, it is considered that much random variability ("noise") was associated with their computation. As they play no significant part in the description of the dental arch form they can be ignored.

The standard error of the method derived for the arch dimensions intercanine width, intermolar width and arch circumference, averaged 0.310 mm and ranged from 0.177 to 0.445 mm, indicating a high degree of reproducibility.

DISCUSSION

For a statistical comparison of dental arch forms to be valid, the quantitative representations of those arch forms must describe them with precision. Stepwise polynomial equations were fit to the data in this study so that independent coefficients, i.e., the coefficients of the highest terms, could be generated for the purposes of statistical comparison. In addition, polynomials have been shown to be quite accurate in translating the curve of the dental arch into a mathematical formula[64,65].

Pepe[65] asserted in her study that the sixth degree polynomial "afforded a significant increase in accuracy of fit over a fourth degree polynomial equation," but she never showed that the differences were statistically significant. The coefficients of determination calculated in our study indicate that the second degree polynomial equations accounted for 96% of the variation in the arch curves, and carrying the polynomials through to the fourth degree increased the accuracy of fit by only 2%. Since the fourth degree polynomials accounted for an average of 98% of the variance encountered, we thus feel that they provided an appropriate measure to describe arch forms.

The first degree polynomial accounts for the linear component of the curve and is shown to be an almost negligible factor; so, too, is the third degree term. In this instance the coefficient of the highest power of x (B_{3_c}) describes arch asymmetry, which was minimal in this sample. The second degree polynomial, however, accounts for the general parabolic shape of the arch and contributes a preponderance of the curve description. The fourth degree equation increases the coefficient of determination only 2% and the coefficient of x^4, which describes the tendency of the curve to be more or less peaked, simply modifies the almost fully defined curve. (If the fourth degree term were large, the arch form would be more pointed; if it were small, the arch shape would be broader anteriorly.) The coefficient of x^2 in the second degree polynomial, then, would appear to be the variable to which the most weight should be attached in considering the within-pair variances (mean square errors) among the three groups.

The F ratios calculated for the within-pair variances in the upper arches show no significant differences among the groups for any of the coefficients. It is noteworthy that for B_{2_b}, the mean square error for all three groups is similar. Not unexpectedly the monozygotic group shows the lowest mean square error, but the dizygotic and sibling groups are only slightly larger. Thus in comparing the variances for B_{2_b} - the coefficient largely responsible for describing general arch form - it appears that the shape of the maxillary arch is amenable to environmental change and not under strict genetic control.

This finding is consistent with the clinical impression that the upper arch is susceptible to many common environmental influences such as thumb-sucking, tongue-thrusting, and lip entrapment. As further confirmation of this impression, the literature reveals several cases of monozygotic twins presenting with obvious differences in their maxillary dental arches. This was reported by Becker[53] and Sakuda[52], cited earlier, and by others such as Leech[67], who treated female monozygotic twins one of whom was Class II, division 1 and the other Class II, division 2.

The converse of maxillary arch vulnerability to environmental forces is the concept that the mandibular arch is much less amenable to changes from these non-genetic influences. Probably the most common derivative of this lower arch corollary is the expectation of relapse when lower canines are expanded by the orthodontist. McCauley[68] and Strang[69] were strong advocates of maintaining the original intercanine dimension, and Riedel in subscribing to the axiom that generally "the arch form of the mandibular arch cannot be permanently altered by appliance therapy"[70] found this to be particularly true of intercanine width. If it is true that mandibular arch form as a whole is less amenable to environmental change than that of the upper, then the within-pair variances calculated for the lower arch in this study would, as stated in the research hypothesis, be expected to appear significantly smaller in the monozygotic group than in the dizygotic or sibling groups.

Upon inspection of the variances for arch form, the mean square errors do bear this out for B_{2_b}. The calculated F ratios indicate that the within-pair variance of the monozygotic group is significantly less than the within-pair variances of the dizygotic or sibling groups, but no differences were found between dizygotic and sibling groups. This suggests the presence of genetic control in determining the form of the mandibular arch and, conversely, the relative lack of environmental influence.

One other significant F value appears in the lower arch comparisons, where the variances of B_{4_d} in the monozygotic group were significantly less than that of the dizygotic. Although this is not unexpected, the failure of the monozygotic mean square error to be significantly lower than that of the sibling group confuses the interpretation. Because the fourth degree coefficient represents a modification of the curve, accounting for only an additional 2% of its variability - and because it reflects only one facet of the entire arch form, a tendency toward being peaked - it is difficult to know what to expect of the F ratios for this set of coefficients. The mean square error of B_{4_d} in the monozygotic sample is, in fact, smaller than that of the sibling group, but the F ratio of 1.64 falls short of the critical F value (2.12). Its failure to achieve significance could be due to the fact that the sample is not truly random and to the smaller size of the dizygotic group leaving it more open to influence by extreme values. But the importance of the B_{4_d} data could be considered equivocal anyway in view of the contribution of that term to the overall description of arch form.

The presence of genetic control in lower arch form, already substantially indicated by the variance ratios for B_{2_b} is given further verification by the statistical comparison of intercanine and intermolar widths in the lower arch. Within-pair variance in these dimensions was found to be significantly smaller in the monozygotic sample than within-pair variances for the dizygotic and sibling groups, indicating again that genetic control is in evidence in the lower arch along with a corresponding lack of environmental influence. This supports, too, the common clinical impression stated by Riedel[70] that the lower dental arch is not permanently tolerant of change at the hands of the orthodontist, himself a potent environmental force.

In contrast to the findings in the lower arch, the within-pair variances of intercanine and intermolar widths in the upper arch failed to achieve statistical significance in both monozygotic/dizygotic and monozygotic/sibling ratios. This reinforces the results of the maxillary arch form comparisons, in which the within-pair variance of the monozygotic group was also not significantly different from that of the dizygotic and sibling samples. However, the F ratio for the sibling to monozygotic variances in intermolar width was extremely close to its critical F value. This might suggest that the posterior portion of the maxillary arch is more resistant to environmental change than the anterior - not unreasonable in view of the effects that most common environmental forces have on the anterior part of the arch. But the dizygotic/monozygotic ratio, on the other hand, does not show this tendency toward significance. Abiding strictly by the results of our statistical tests, then, it can be said that the maxillary intercanine and intermolar widths in this sample do not demonstrate the presence of detectable genetic control and suggest instead a susceptibility to environmental influences.

The data of Menezes et al[51] agree with these findings in the maxillary arch, but, again, their small sample size limits the validity of their conclusions. Lundstrom in his study of 202 sets of twins found more within-pair variation of upper molar width in the dizygotic sample than in the monozygotic, but he did not test the differences statistically[71]. Shapiro, in a twin study of palatal dimensions, determined palatal width as the distance between the first molars, a measure equivalent to the maxillary intermolar width used in this study. He concluded that for this dimension the "heredity component of variation ... was not strong"[32] although in females the dizygotic variation was significantly greater than monozygotic variation. In a study using similar measurements, Riquelme and Green found that palatal width in their sample "showed a very strong hereditary variation."[33] Assuming that palatal width as measured in these studies is comparable to the intermolar dimension used here, our findings tend to support those of Shapiro rather than Riquelme and Green.

Of all the variance ratios calculated for the upper arch, the only ones to be statistically significant were the arch circumference values. The within-pair variance of this dimension was found to be significantly smaller in the monozygotic sample than in both dizygotic and sibling groups. Arch circumference would appear to be largely a function of tooth dimension, which Lundstrom has shown to be under strong genetic control, and of the extent to which spacing and crowding prevail in the arch. Since Lundstrom's study also "indicates the significance of heredity to the origin of crowding and spacing,"[17] it is not surprising that the circumference variation of the monozygotic upper arches was significantly less than that of the other two groups. Even though maxillary arch form showed itself to be under the influence of environmental factors, the added genetic components of tooth dimension and spacing/crowding appear to have tipped the balance toward genetic control in arch circumference.

As would be expected from the foregoing data, the arch circumference F ratios of the mandibular arch proved to be significant for both dizygotic/monozygotic and sibling/monozygotic comparisons. The presence of genetic control already demonstrated in lower arch form apparently enhanced the genetic components of tooth dimension and spacing/crowding to enable genetic influences to hold sway in a convincing fashion.

The rationale of the interpretation above is based upon the traditional genetic hypothesis of the twin method. Because monozygotic twins are assumed to have identical genetic constitutions, any variation measured within pairs is attributed to environmental factors and measurement error. If environment has a minimal effect on the trait being quantified, the variance should be small; if environment plays a dominant role in determining the trait, the variance should approach that found in like-sexed dizygotic twins.

Variance within pairs of like-sexed dizygotic twins is attributed to environmental factors, measurement error, and to genetic variation as well. Genetic variation within pairs of these twins should be equivalent to that in like-sexed siblings. Total variation, however, should be slightly less in the dizygotic twins because of their presumably similar intrauterine environment and the possibility of a more comparable postnatal environment. Measurement error is assumed to show no bias toward any of the three groups, and postnatal environment is assumed to be similar within pairs of both types of twins.

Thus the hierarchy of within-pair variation where genetic influences predominate would find monozygotic twins showing less variation than the dizygotic twins and the dizygotic variation approximating but measuring slightly less than that of the sibling pairs. If environmental forces were dominant, the monozygotic variation would approximate that of the dizygotic and sibling pairs but still fall short of them due to the likely presence of at least some genetic control. Because of these assumptions, one way variance analysis was used in this study with the group expected to have less variation forming the denominator of the variance ratio.

In the earlier twin studies it was common to calculate a heritability estimate. Heritability was considered to be the percentage of total dizygotic variance contributed only by the genetic component. The latter term was computed by factoring out the environmental influence, i.e., the monozygotic variance[72]:

$$\text{Heritability} = \frac{\text{dizygotic variance} - \text{monozygotic variance}}{\text{dizygotic variance}}$$

Osborne and DeGeorge[22] considered this figure to be of questionable validity and felt it contributed nothing of value that simple variance analysis could not provide equally as well. In addition, the variance ratios required fewer statistical assumptions, a definite advantage in a method beset by a large number of assumptions.

Aside from recognizing the assumptions underlying twin studies, an interpretation of twin data must also take into account the true nature of what is being measured. The genotypic variance measured by the twin method includes additive, dominance, and epistatic genetic effects, referred to as heritability in the broad sense, and because only the additive effects are directly transmitted from parent to offspring, twin studies cannot determine to what extent a trait is inherited. In the case of polygenic characters, heritability refers to this transmission of parental traits to offspring through the genes, whereas the term genetic control signifies the variation of an individual due to his genotype. In this context, then, it is not appropriate to infer estimates of heritability from twin studies which are assessing polygenic control of variance[73].

It is important, too, to recognize that when polygenic control of variance is being measured, one significant factor not accounted for is the interaction of genotype and environment. The traditional twin study appears to be a reflection of the original nature-nurture question, and, as Horowitz and Hixon noted several years ago, "this dichotomy, heredity or environment, is a misleading framework within which to consider a complex morphological trait such as malocclusion."[74] Potter[73] felt "the question of genotype-environment interaction can best be approached by twin studies," but that "geneticists ... have not optimally explored the potentials of twin research methods." The recent development of multivariate techniques for analysis on twins, she added, will make it possible to explore the interaction question in more detail. Osborne and DeGeorge, on the other hand, asserted that because monozygotic twins are identical for their whole genetic constitutions, they should respond in a similar way to specific environmental influences[22]. Perhaps the question will be answered with certainty as more sophisticated techniques become available for twin analysis; the experimental model in our study, however, does not attempt to account for the effects of genotype-environment interaction.

The limitations of the twin method in general, including its biases, both biological and statistical, and the assumptions which must be made, have been discussed, but areas of potential criticism found in this particular study must yet be recognized. The first has to do with our sample. We would have preferred the dizygotic group to have been as large as the monozygotic and sibling groups, but we feel it was large enough to yield valid data. The fact that five dizygotic pairs were obtained from another area could indicate a source of variation. But these individuals were also of northwest European ancestry. It is not, however, possible to speculate what the influence of these individuals being from triplet sets has on the variability of our dizygotic data.

Ideally all the dental arches we were dealing with would have been mature, stable ones. The latter condition was not always completely satisfied, as mentioned in the Materials and Methods, but the ages of the individuals within each pair were, with few exceptions, very close to one another. We feel confident, then, that none of these limitations has compromised our results.

In considering ideal situations, one could also insist that a three dimensional model would offer a more accurate representation of arch form but this would have necessitated a much more complicated model.

Orthodontists, are, of course, interested in the form of the dental arches in the other dimension, e.g., the curve of Spee or the curve of Wilson, but almost always when the shape of the arch is being considered, the conceptualized form is the two dimensional curve.

The variables used to represent these arch forms in our statistical comparisons might give rise to one further question: whether the coefficients themselves were normally distributed. A histogram of the B_{2_b} coefficient was constructed and no obvious skewness was apparent. Thus the assumption of the normalcy of the distribution was not negated.

While it has been necessary to consider the many limitations of twin studies, and of this twin study in particular, we nevertheless submit that our experimental model has yielded statistical data which are both valid and positive. These data lend quantitative credence to clinical impressions commonly held for the form and dimensions of both upper and lower dental arches. The form of the mandibular arch, as well as its intercanine and intermolar widths, are all under genetic control and do not appear to be subject to the forces of environment. This bears out the frequently observed tendency toward relapse found in the lower dental arch after its form and dimensions, especially the intercanine dimension, have been altered by the orthodontist. Conversely the upper arch shows little evidence of genetic control but appears to be amenable to environmental change. This supports the impression that the orthodontist can alter the maxillary arch with less concern over relapse than would be warranted in changing the mandibular arch.

SUMMARY AND CONCLUSIONS

The purpose of this twin study was to assess the significance of genetic control in the form and certain dimensions of an individual's dental arches. The sample consisted of twenty-two pairs of monozygotic twins, thirteen pairs of like-sexed dizygotic twins, and twenty-one pairs of like-sexed siblings. Zygosity determinations for the twins were primarily based on serological tests. Dentitions which were mutilated or had undergone orthodontic treatment were not included, and to as great an extent as possible, only those which were in the complete permanent dentition and relatively stable were utilized. The ages at which the dental casts were obtained ranged in the monozygotic group from twelve to twenty-four years with a mean of 16.27 years; in the dizygotic group from ten to nineteen years with a mean of 13.87 years, and in the sibling group from ten to twenty-four years with a mean of 17.69 years. Within-pair age differences ranged from one to four years in sixteen pairs, but all these individuals were in their late teens or early twenties.

Plaster casts of these dentitions were marked with a series of points on the buccal cusp tips of the posterior teeth, the cusp tips of the canines and the middle of the incisal edge of the anterior teeth. Also two points on the midpalatal raphe and two on the heels of the casts were used for orientation. After the upper and lower casts of each subject were oriented to one another and their occlusal planes made horizontal, they were individually photographed and enlarged to two and one-half times their original size.

The data points recorded on the photographs were digitized, fed into a computer and then scaled and transformed. The dental arch form of each upper and lower cast was defined as the continuum of its data points, each given an x and y coordinate, and the computer was programmed to fit to each arch form a series of standard polynomial equations, first through fourth degree. Intercanine and intermolar dimensions of each arch were also recorded, as well as arch circumference.

Within-pair variances (mean square error) of the coefficients, independently derived from the stepwise polynomial equations to describe dental arch form, were computed for each arch in each group. Within-pair variances of intercanine and intermolar width, and arch circumference were calculated for each arch in each of the three groups. Variance ratios were then calculated to determine whether monozygotic intrapair variances were significantly less than the respective dizygotic and sibling variances and whether dizygotic and sibling variances were comparable. Error of the method was determined from replicate data of twelve randomly selected cases.

Conclusions drawn from the results of this study were:

1 - The error of the method was a negligible factor contributing to the within-pair variances of the arch dimensions and the coefficients of the polynomial equations fit to the dental arch form.

2 - The coefficients of the polynomial equations which contributed most to the fit of the dental arches were the second and fourth degree terms. The lack of importance of the first and third degree terms indicated that there was very little asymmetry in the dental arches examined.

3 - The highest degree polynomial equation fit to the data points was the fourth degree. This accounted for an average of 98% of the variation in both maxillary and mandibular arches and was considered sufficiently accurate to adequately describe dental arch form.

4 - The intrapair variance of the coefficients which were primary factors in describing mandibular arch form was significantly lower in the monozygotic group than in the dizygotic and sibling groups. Thus genetic factors were more influential than environmental forces in determining mandibular arch form.

5 - Intercanine and intermolar dimensions in the mandibular arch were also little influenced by environmental factors in the monozygotic group. But the larger intrapair variances in the dizygotic and sibling groups were primarily due to genetic differences. Hence these dimensions in the mandibular arch were under considerable genetic control.

6 - Lack of significant difference in the intrapair variance of the coefficients which primarily described maxillary arch form, indicates that these arches were susceptible to change by environmental influences and were not under strong genetic control.

7 - Intercanine and intermolar widths in the maxillary arch were also not under strong genetic control and were influenced by environmental factors.

8 - Significantly lower within-pair variances in maxillary and mandibular arch circumference were found in the monozygotic group compared to the dizygotic and sibling groups. Arch circumference, which is a function of tooth size and crowding or spacing, was under strong genetic control.

9 - These results may be considered to support the clinical

impression gained by orthodontists that while the maxillary dental arch is amenable to permanent change, the mandibular arch form is not able to be changed to any great degree, especially across the canines, and remain stable.

BIBLIOGRAPHY

1. M'Quillen, J.H.: "Hereditary transmission of dental irregularities," Dental Cosmos 12:27-29, 73-75, 193-195, 1870.
2. Huskins, C.L.: "On the inheritance of an anomaly of human dentition," J. of Heredity 21:279-282, 1930.
3. Keeler, C.E., and Short, R.: "Hereditary absence of upper lateral incisors," J. of Heredity 25:391-392, 1934.
4. Keeler, C.E.: "Heredity in dentistry," Dental Cosmos 77:1147-1163, 1935.
5. Mason, R.: "Four generations of harelip," J. of Heredity, 17:52, 1935.
6. Bachrach, F.H., and Young, M.: "A comparison of the degree of resemblance in dental characters shown in pairs of identical and fraternal types," Br. Dent. J. 48:1293-1304, 1927.
7. Goldberg, S.: "Biometrics of identical twins from the dental viewpoint," J. Dent. Res. 9:363-409, 1929.
8. Korkhaus, G.: "Anthropologic and odontologic studies of twins," International J. of Orthodontia, Oral Surgery and Radiography 16:640-467, 1930.
9. Iwagaki, H.: "Hereditary influence of malocclusion," Am. J. of Orthod. and Oral Surgery 24:328-336, 1938.
10. Rubbrecht, O.: "A study of the heredity of the anomalies of the jaws," Am. J. of Orthod. and Oral Surgery 25:751-759, 1939.
11. Johnson, A.L.: "The constitutional factor in skull form and

dental occlusion," Am. J. of Orthod. and Oral Surgery 26:627-663, 1940.

12. Hughes, B.O., and Moore, G.R.: "Heredity, growth, and the dento-facial complex," Angle Orthod. 11:217-222, 1941.

13. Hughes, B.O.: "Heredity as a factor in cranial and facial development," Am. J. of Orthod. and Oral Surgery 28:357-360, 1942.

14. Wylie, W.L.: "A quantitative method for the comparison of craniofacial patterns in different individuals," Am. J. of Anatomy 74:39-60, 1944.

15. Cohen, J.T., Oliver, C.P., and Bernick, S.: "Dental studies of triplets," J. Dent. Res. 21:413-420, 1942.

16. Lundstrom, A.: "The importance of genetic and non-genetic factors in the facial skeleton studied in 100 pairs of twins," Eur. Orthod. Soc. Rep. Cong. 30:92-107, 1954.

17. Lundstrom, A.: "The significance of genetic and non-genetic factors in the profile of the facial skeleton," Am. J. Orthod. 41:910-916, 1955.

18. Lundstrom, A.: Tooth Size and Occlusion in Twins, New York, 1948, S. Karger.

19. Walker, N.F.: "Review of Tooth Size and Occlusion in Twins by A. Lundstrom," Am. J. of Human Genetics 3:77, 1951.

20. Horowitz, S.L., Osborne, R.H., and DeGeorge, F.V.: "A cephalometric study of craniofacial variation in adult twins," Angle Orthod. 30:1-5, 1960.

21. Hunter, W.S.: "A study of the inheritance of craniofacial characteristics as seen in lateral cephalograms of 72 like-sexed twins," Trans. Eur. Orthod. Soc. 41:59-70, 1965.

22. Osborne, R.H., and DeGeorge, F.V.: Genetic Basis of Morphologic Variation, Cambridge, 1959, Harvard University Press.

23. Snodgrasse, R.: "A family line study in cephalofacial growth," Am. J. of Orthod. and Oral Surgery 34:714-724, 1948.

24. Stein, K.F., Kelly, T.J., and Wood, E.: "Influence of heredity in the etiology of malocclusion," Am. J. Orthod. 42:125-141, 1956.

25. Noyes, H.J.: "A review of the genetic influence on malocclusion," Am. J. Orthod. 44:81-98, 1958.

26. Curtner, R.M.: "Predetermination of the adult face," Am. J. Orthod. 39:201-217, 1953.

27. Kraus, B.S., Wise, W.J., and Frei, R.H.: "Heredity and the craniofacial complex," Am. J. Orthod. 45:172-217, 1959.

28. Moorrees, C.F.A.: "Genetic considerations in dental anthropology," in Witkop, C.J., editor: Genetics in Dental Health, New York, 1962, McGraw-Hill.

29. Goodman, H.O.: "Genetic parameters of dentofacial development," J. Dent. Res. 9:363-409, 1929.

30. Watnick, S.S.: "Inheritance of craniofacial morphology," Angle Orthod. 42:339-351, 1972.

31. Arya, B.S., Savara, B.S., Clarkson, Q.D., and Thomas, D.R.: "Genetic variability of craniofacial dimensions," Angle Orthod. 43:207-215, 1973.

32. Shapiro, B.L.: "A twin study of palatal dimensions partitioning genetic and environmental contributions to variability," Angle Orthod. 39:139-151, 1969.

33. Riquelme, A., and Green, L.J.: "Palatal width, height, and length in human twins," Angle Orthod. 40:71-79, 1970.

34. Rebich, T., and Markovic, M.: "Inheritance of tooth dimensions: a quantitative genetic study," Trans. Eur. Orthod. Soc. 52:59-70, 1976.

35. Niswander, J.D.: "Genetics of common dental disorders," Dent. Clin. North Am. 19:197-206, 1975.

36. Harris, J.E., Kowalski, C.J., and Walker, S.J.: "Intrafamilial dentofacial associations for Cl. II division 1 problems," Am. J. Orthod. 67:563-570, 1975.

37. Litton, S.F., Ackerman, L.V. Isaacson, R.J., and Shapiro, B.L.: "A genetic study of Cl. III malocclusion," Am. J. Orthod. 58: 565-577, 1970.

38. Harris, J.E., Kowalski, C.J., and Walker, S.J.: "Dentofacial differences between "normal" sibs of Cl. II and Cl. III patients," Angle Orthod. 45:103-107, 1975.

39. Nelson, M.M.: "A brief study of the genetics of malocclusion," Trans. Eur. Orthod. Soc. 45:55-68, 1969.

40. Eismann, D.: "Heredity of malocclusion assessed by means of family investigations," Trans. Eur. Orthod. Soc. 52:71-76, 1976.

41. Horowitz, S.L., and Osborne, R.H.: "The genetic aspects of craniofacial growth," in Moyers, R.E., and Krogman, W.M., editors: Craniofacial Growth in Man, New York, 1971, Pergamon Press, 183-192.

42. Fisher, R.A.: "The correlation between relatives on the supposition of Mendelian inheritance," Trans. Royal Soc. Edin. 52: 399-433, 1918.

43. Price, B.: "Primary biases in twin studies," Am. J. Human Genetics 2:293-352, 1950.

44. Newman, H.H., Freeman, F.N., and Holzinger, K.J.: Twins: A Study

of Heredity and Environment, Chicago, 1937, Chicago University Press.

45. Stern, C.: Principles of Human Genetics, San Francisco, 1960, W.H. Freeman Co., 522-573.

46. Kempthorne, O., and Osborne, R.H.: "The interpretation of twin data," Am. J. Human Genetics 13:320-339, 1961.

47. Allen, G.: "Twin research: problems and prospects," Progress in Med. Genet. 41:242-269, 1965.

48. Newton, S.D.: "On the mouths of triplets," Dental Record 57: 364-368, 1937.

49. Braun, M.L.: "Likeness of dental arches and of persons without blood relationship," J. Am. Dent. Assoc. 25:1093-1098, 1938.

50. Cohen, J.T., Oliver, C.P., and Bernick, S.: "Dental studies of triplets," J, Dent. Res. 21:233-239, 1942.

51. Menezes, D., Foster, T., and LaVelle, C.: "Genetic influences on dentition and dental arch dimensions: a study of monozygotic and dizygotic triplets," Am. J. Phys. Anthropol. 40:213-220, 1974.

52. Sakuda, M., Matsumoto, M., Yoshida, K., and Takimoto, K.: "Monozygotic twins showing differences in occlusion," Angle Orthod. 43: 41-52, 1973.

53. Becker, A.: "Monozygosity in twins: a detailed investigation," Am. J. Orthod. 72:65-75, 1977.

54. Black, G.V.: Descriptive Anatomy of the Human Teeth, Philadelphia, 1902, S.S. White Dental Manufacturing Co., 130-152.

55. Angle, E.H.: Treatment of Malocclusion of the Teeth, Philadelphia, 1907, S.S. White Dental Manufacturing Co., 23.

56. Sicher, H.: Oral Anatomy, St. Louis, 1952, C.V. Mosby Co., 262.

57. Webster's New International Dictionary, 2nd edition, s.v. "catenary."

58. MacConaill, M.A., and Scher, E.A.: "The ideal form of the human dental arcade," Dental Record 69:285-302, 1949.

59. Scott, J.H.: "The shape of the dental arches," J. Dent. Res. 36: 996-1001, 1957.

60. Burdi, A.R.: "Morphogenesis of mandibular dental arch shape in human embryos," J. Dent. Res. 47:50-58, 1968.

61. Currier, J.H.: "A computerized geometric analysis of human dental arch form," Am. J. Orthod. 56:165-175, 1969.

62. Brader, A.C.: "Dental arch form related with intraoral forces," Am. J. Orthod. 61:541-561, 1972.

63. Biggerstaff, R.H.: "Computerized dentition analyses and simulations," in Cook, T.J., editor: Proceedings of the Third Orthodontic Congress, St. Albans, England, 1975, Crosby, Lockwood, Staples.

64. Lu, K.H.: "An orthogonal analysis of the form, symmetry and asymmetry of the dental arch," Archives of Oral Biology 11:1057-1069, 1966.

65. Pepe, S.H.: "Polynomial and catenary curve fits to human dental arches," J. Dent. Res. 54:1124-1132, 1975.

66. Moyers, R.E., Van der Linden, F., Riolo, M.L., and McNamara, J.A.: Standards of Human Occlusal Development, Ann Arbor, 1976, University of Michigan, 5-26.

67. Leech, H.L.: "Angle's Class II, division 1, and Class II, division 2, in identical twins," Dental Practitioner and Dental Record 5: 341-345, 1955.

68. McCauley, D.R.: "The cuspid and its function in retention," Am. J. Orthod. 30:196-205, 1944.

69. Strang, R.H.W.: "Factors of influence in producing a stable result in treatment of malocclusions," Am. J. Orthod. 32:313-332, 1946.

70. Riedel, R.: "A review of the retention problem," Angle Orthod. 30: 179-199.

71. Lundstrom, A.: "An investigation of 202 pairs of twins regarding fundamental factors in the aetiology of malocclusion," Dental Record 69:251-264, 1949.

72. Holzinger, K.: "The relative effect of nature and nurture influences on twin differences," J. Educ. Psychol. 20:241-248, 1929.

73. Potter, R.H.Y.: "The genetics of tooth size," in Stewart, R.E., and Prescott, G.H., editors: Oral Facial Genetics, St. Louis 1966, C.V. Mosby Co.

74. Horowitz, S.L., and Hixon, E.H.: The Nature of Orthodontic Diagnosis, St. Louis, 1966, C.V. Mosby Co.

TABLE I

Average Coefficients of Determination Obtained for the First, Second, Third and Fourth Degree Polynomial Equations

ARCH	First Degree		Second Degree		Third Degree		Fourth Degree	
	Mean	Range	Mean	Range	Mean	Range	Mean	Range
UPPER	.004	.00-.09	.957	.87-.99	.961	.87-.99	.979	.89-1.00
LOWER	.006	.00-.09	.962	.84-.99	.969	.84-1.0	.982	.89-1.00

TABLE 2

Analysis of the Coefficients of the Polynomial Equations Fitted to the Maxillary Arch

A. Within-pair Mean Square Errors of the Coefficients

Group N =	Monozygotic 22	Dizygotic 13	Siblings 21
B_{1a}	3.30×10^{-3}	1.98×10^{-3}	0.52×10^{-3}
B_{2b}	2.27×10^{-7}	3.20×10^{-7}	3.25×10^{-7}
B_{3c}	8.35×10^{-12}	7.30×10^{-12}	3.74×10^{-12}
B_{4d}	2.99×10^{-16}	4.60×10^{-16}	2.11×10^{-16}

B. F Ratios*

	$\frac{\text{Dizygotic}}{\text{Monozygotic}}$	$\frac{\text{Sibling}}{\text{Monozygotic}}$	$\frac{\text{Sibling}}{\text{Dizygotic}}$
B_{1a}	0.60	0.15	0.26
B_{2b}	1.41	1.43	1.01
B_{3c}	0.87	0.45	0.51
B_{4d}	1.54	0.70	0.46

* None of the F ratios is significant at the .05 level.

TABLE 3

Analysis of the Coefficients of the Polynomial Equations Fitted to the Mandibular Arch

A. Within-pair Mean Square Errors of the Coefficients

Group N =	Monozygotic 22	Dizygotic 13	Siblings 21
B_1 a	6.23×10^{-3}	3.34×10^{-3}	0.75×10^{-3}
B_2 b	1.56×10^{-7}	7.50×10^{-7}	4.80×10^{-7}
B_3 c	1.38×10^{-11}	2.36×10^{-11}	0.89×10^{-11}
B_4 d	2.60×10^{-16}	8.16×10^{-16}	4.27×10^{-16}

B. F Ratios

	$\frac{\text{Dizygotic}}{\text{Monozygotic}}$	$\frac{\text{Sibling}}{\text{Monozygotic}}$	$\frac{\text{Sibling}}{\text{Dizygotic}}$
B_1 a	.54	.12	.22
B_2 b	4.80***	3.07*	.64
B_3 c	1.71	.64	.38
B_4 d	3.14**	1.64	.52

* $.005<p<.01$

** $.01<p<.025$

*** $.001<p<.005$

All other F ratios are nonsignificant at the .05 level.

TABLE 4

Analysis of the Distance Variables
Measured on the Maxillary Arch (in mm.)

A. Within-pair Mean Square Errors

	Monozygotic		Dizygotic		Siblings	
Variable	N		N		N	
Intercanine Width	21	2.147	12	3.466	21	2.777
Intermolar Width	22	3.058	13	2.775	21	6.336
Arch Circumference	22	2.640	13	7.234	20	16.390

B. F Ratios

	Dizygotic / Monozygotic	Sibling / Monozygotic	Sibling / Dizygotic
Intercanine Width	1.61	1.29	0.80
Intermolar Width	0.91	2.07*	2.28
Arch Circumference	2.74**	6.20***	2.26

* p=.05
** .01<p<.025
*** p<.005
All other F ratios are nonsignificant at the .05 level.

TABLE 5

Analysis of the Distance Variables
Measured on the Mandibular Arch (in mm.)

A. Within-pair Mean Square Errors

	Monozygotic		Dizygotic		Siblings	
Variable	N		N		N	
Intercanine Width	22	0.771	13	3.746	20	2.994
Intermolar Width	22	1.615	12	6.984	21	8.452
Arch Circumference	22	3.057	12	16.597	21	9.659

B. F Ratios

	Dizygotic / Monozygotic	Sibling / Monozygotic	Sibling / Dizygotic
Intercanine Width	4.86**	3.88**	0.78
Intermolar Width	4.32**	5.23**	1.21
Arch Circumference	5.43**	3.15*	0.58

* 01<p<05
** p<.005
All other F ratios are nonsignificant at the .05 level.

Appendix 1A

Coefficients of the Highest Power of X Derived from Stepwise Polynomial Equations
Maxillary Arch

MONOZYGOTIC TWINS

Subject	B_{1} a	B_{2} b	B_{3} c	B_{4} d	Overall* R^2	Overall* F
43.0	$-8.6679x10^{-3}$	$5.0816x10^{-3}$	$-1.5844x10^{-6}$	$1.1634x10^{-8}$	.99	339.28
44.0	$-2.0853x10^{-2}$	$7.5148x10^{-3}$	$-8.7441x10^{-6}$	$5.3712x10^{-8}$	.97	117.10
62.2	$1.4969x10^{-2}$	$4.6588x10^{-3}$	$1.8488x10^{-6}$	$2.6241x10^{-8}$	.99	493.87
62.3	$2.9791x10^{-2}$	$5.5015x10^{-3}$	$1.5188x10^{-6}$	$4.4899x10^{-8}$	.99	254.89
71.0	$-9.4745x10^{-2}$	$6.6669x10^{-3}$	$1.0998x10^{-6}$	$6.2699x10^{-8}$	1.00	1066.58
72.0	$1.1974x10^{-2}$	$7.0031x10^{-3}$	$9.7402x10^{-7}$	$7.2487x10^{-8}$	.98	196.34
76.0	$-2.1106x10^{-2}$	$5.4629x10^{-3}$	$-3.2528x10^{-6}$	$3.2729x10^{-8}$	.97	95.09
77.0	$2.8810x10^{-2}$	$5.0397x10^{-3}$	$-3.9209x10^{-9}$	$2.9827x10^{-8}$	.98	192.03
83.1	$3.9246x10^{-2}$	$6.2316x10^{-3}$	$2.3821x10^{-6}$	$4.9425x10^{-8}$	.99	521.07
83.2	$-4.5555x10^{-2}$	$6.1442x10^{-3}$	$-5.9516x10^{-6}$	$3.2440x10^{-8}$	.99	302.77
100.1	$4.4937x10^{-2}$	$4.6228x10^{-3}$	$2.4312x10^{-6}$	$2.2070x10^{-8}$	1.00	727.14
100.2	$-4.5480x10^{-2}$	$4.4988x10^{-3}$	$-1.0466x10^{-6}$	$2.7594x10^{-8}$	.96	61.47
109.1	$4.6077x10^{-3}$	$5.7140x10^{-3}$	$1.5714x10^{-6}$	$2.7425x10^{-8}$	.95	61.03
109.2	$8.6161x10^{-3}$	$5.6968x10^{-3}$	$-6.5271x10^{-7}$	$4.4187x10^{-8}$	.96	69.01
119.1	$-1.3989x10^{-2}$	$5.8994x10^{-3}$	$-1.5850x10^{-8}$	$3.6793x10^{-8}$	.98	163.32
119.2	$8.4062x10^{-2}$	$5.7669x10^{-3}$	$3.5347x10^{-7}$	$1.2935x10^{-8}$	.91	34.54
121.1	$-3.8073x10^{-2}$	$4.9911x10^{-3}$	$-4.0274x10^{-6}$	$2.0118x10^{-8}$	.99	356.47
121.2	$1.1852x10^{-2}$	$4.6544x10^{-3}$	$2.2414x10^{-6}$	$1.3271x10^{-9}$	.99	463.28
121.3	$-1.0595x10^{-2}$	$5.5951x10^{-3}$	$1.7406x10^{-6}$	$4.7426x10^{-8}$	.99	266.58
121.4	$-2.8668x10^{-2}$	$5.3858x10^{-3}$	$-1.0847x10^{-6}$	$5.1973x10^{-8}$	.98	210.19
132.0	$-2.2653x10^{-1}$	$5.6235x10^{-3}$	$-3.4312x10^{-6}$	$3.9814x10^{-8}$	.98	120.25
133.0	$-1.4620x10^{-2}$	$5.6539x10^{-3}$	$2.5071x10^{-6}$	$4.0490x10^{-8}$	.98	155.74

Appendix 1A (continued)

Subject	B_{1_a}	B_{2_b}	B_{3_c}	B_{4_d}	Overall* R^2	Overall* F
143.0	$-3.4261x10^{-2}$	$4.8662x10^{-3}$	$-9.6561x10^{-7}$	$4.4719x10^{-8}$	.98	204.68
144.0	$-1.4978x10^{-2}$	$5.3760x10^{-3}$	$-1.2205x10^{-6}$	$2.8188x10^{-8}$	.98	168.61
155.1	$-1.6337x10^{-2}$	$5.8847x10^{-3}$	$-1.2552x10^{-6}$	$4.5865x10^{-8}$	.98	149.05
155.2	$-2.2947x10^{-2}$	$5.5167x10^{-3}$	$-1.2146x10^{-7}$	$3.9726x10^{-8}$	.95	57.19
165.0	$1.5138x10^{-2}$	$6.8129x10^{-3}$	$-1.1388x10^{-6}$	$1.4217x10^{-7}$	.95	39.73
167.0	$-6.1502x10^{-2}$	$6.7208x10^{-3}$	$2.7508x10^{-6}$	$6.4781x10^{-8}$	.97	73.91
183.1	$-2.8362x10^{-2}$	$5.0260x10^{-3}$	$-3.6309x10^{-6}$	$3.6506x10^{-8}$	.96	87.39
183.2	$-1.8213x10^{-3}$	$5.9312x10^{-3}$	$-2.1873x10^{-6}$	$3.7167x10^{-8}$	.97	115.49
242.1	$-9.5155x10^{-2}$	$6.2523x10^{-3}$	$-1.0577x10^{-6}$	$5.6867x10^{-8}$	.96	77.70
242.2	$-2.8211x10^{-2}$	$6.1015x10^{-3}$	$-2.5253x10^{-7}$	$5.0418x10^{-8}$	.96	78.78
244.1	$2.3493x10^{-2}$	$6.6076x10^{-3}$	$8.3972x10^{-7}$	$6.7590x10^{-8}$	.97	114.22
244.2	$-5.1367x10^{-2}$	$6.8753x10^{-3}$	$-8.9409x10^{-6}$	$4.4766x10^{-8}$	.93	39.80
248.1	$-3.1505x10^{-2}$	$5.9862x10^{-3}$	$-2.6419x10^{-6}$	$4.6731x10^{-8}$	.96	89.07
248.2	$9.4428x10^{-2}$	$5.8082x10^{-3}$	$-5.1580x10^{-6}$	$5.1174x10^{-8}$	.97	102.01
250.1	$-2.3805x10^{-2}$	$5.1337x10^{-3}$	$-4.0273x10^{-7}$	$2.7697x10^{-9}$	.98	139.81
250.2	$8.4150x10^{-5}$	$5.8089x10^{-3}$	$-8.6642x10^{-7}$	$2.6224x10^{-8}$	.98	212.84
251.1	$-1.9316x10^{-2}$	$5.5724x10^{-3}$	$4.3679x10^{-7}$	$4.5342x10^{-8}$	1.00	753.61
251.2	$2.5031x10^{-3}$	$4.6613x10^{-3}$	$-7.9921x10^{-7}$	$1.3186x10^{-8}$	.99	426.41
290.1	$-3.6230x10^{-2}$	$6.5399x10^{-3}$	$-6.0686x10^{-6}$	$4.7483x10^{-8}$	.99	397.47
290.2	$-2.0606x10^{-1}$	$6.2720x10^{-3}$	$-3.6627x10^{-6}$	$7.5009x10^{-8}$	1.00	666.31
317.1	$-3.5093x10^{-2}$	$4.5867x10^{-3}$	$-4.4820x10^{-6}$	$1.7237x10^{-8}$	.98	135.99
317.2	$-3.7659x10^{-2}$	$5.0104x10^{-3}$	$-1.6926x10^{-6}$	$5.1401x10^{-9}$	.98	167.76
Mean	-2.4×10^{-2}	$5.699x10^{-3}$	$-1.221x10^{-6}$	$4.039x10^{-8}$		

* Due to fourth degree polynomial equation

Appendix 1B

Coefficients of the Highest Power of X Derived from Stepwise Polynomial Equations
Mandibular Arch

MONOZYGOTIC TWINS

Subject	B_{1_a}	B_{2_b}	B_{3_c}	B_{4_d}	Overall* R^2	Overall* F
43.0	$-2.7468x10^{-2}$	$5.4512x10^{-3}$	$-5.4976x10^{-7}$	$-6.2783x10^{-10}$	.99	246.63
44.0	$-2.7923x10^{-2}$	$5.6495x10^{-3}$	$-4.7390x10^{-6}$	$1.6099x10^{-8}$	.99	250.59
62.2	$1.4930x10^{-2}$	$5.1526x10^{-3}$	$2.1181x10^{-6}$	$3.5664x10^{-8}$	1.00	723.61
62.3	$4.5087x10^{-2}$	$4.6754x10^{-3}$	$5.1495x10^{-6}$	$3.3920x10^{-8}$	.99	379.95
71.0	$-3.8397x10^{-2}$	$7.3329x10^{-3}$	$4.1112x10^{-6}$	$4.7234x10^{-8}$	.99	418.78
72.0	$3.2607x10^{-2}$	$7.5922x10^{-3}$	$2.6713x10^{-6}$	$8.0589x10^{-8}$	.99	580.26
76.0	$-2.0240x10^{-2}$	$6.3409x10^{-3}$	$-3.1265x10^{-6}$	$4.7758x10^{-8}$	.98	185.68
77.0	$8.8450x10^{-3}$	$5.9887x10^{-3}$	$-1.2539x10^{-6}$	$3.7019x10^{-8}$	.99	482.49
83.1	$4.3597x10^{-2}$	$7.2332x10^{-3}$	$3.0509x10^{-6}$	$9.1915x10^{-8}$	.98	179.28
83.2	$2.1107x10^{-2}$	$6.7554x10^{-3}$	$-8.7398x19^{-6}$	$8.7387x10^{-8}$	.98	157.14
100.1	$4.4720x10^{-2}$	$5.2504x10^{-3}$	$-1.3076x10^{-7}$	$1.1909x10^{-8}$	.99	448.29
100.2	$1.9965x10^{-2}$	$5.2297x10^{-3}$	$-2.5659x10^{-6}$	$1.9475x10^{-3}$	.99	646.41
109.1	$1.2953x10^{-2}$	$6.7765x10^{-3}$	$-5.1002x10^{-7}$	$7.1815x10^{-8}$	.98	181.60
109.2	$8.3505x10^{-3}$	$6.7242x10^{-3}$	$-4.6619x10^{-6}$	$7.2153x10^{-8}$	.99	424.93
119.1	$-1.0839x10^{-2}$	$4.6755x10^{-3}$	$-4.0393x10^{-7}$	$1.8794x10^{-8}$	.99	357.08
119.2	$-7.0683x10^{-2}$	$5.4848x10^{-3}$	$-2.4856x10^{-6}$	$5.3534x10^{-8}$	.95	68.68
121.1	$-6.3927x10^{-2}$	$5.9707x10^{-3}$	$-6.2471x10^{-6}$	$1.0765x10^{-8}$	.99	252.06
121.2	$1.8968x10^{-2}$	$5.9888x10^{-3}$	$4.3517x10^{-7}$	$5.4236x10^{-9}$	.98	189.95
121.3	$3.2911x10^{-2}$	$6.5867x10^{-3}$	$3.9306x10^{-6}$	$3.9306x10^{-6}$	.98	210.16
121.4	$-1.3196x10^{-2}$	$6.1807x10^{-3}$	$-2.6485x10^{-6}$	$7.5233x10^{-8}$	.99	238.65
132.0	$-1.8618x10^{-1}$	$5.8412x10^{-3}$	$-5.1314x10^{-6}$	$6.1976x10^{-8}$	.98	139.23
133.0	$2.3114x10^{-1}$	$6.0163x10^{-3}$	$4.8869x10^{-6}$	$5.3789x10^{-8}$	.99	254.37

Appendix 1B (continued)

Subject	B_{1_a}	B_{2_b}	B_{3_c}	B_{4_d}	Overall* R^2	Overall* F
143.0	$3.9989x10^{-3}$	$5.9708x10^{-3}$	$-2.0726x10^{-6}$	$2.8649x10^{-8}$	.99	511.48
144.0	$-1.4954x10^{-2}$	$6.2051x10^{-3}$	$-2.0816x10^{-6}$	$1.9598x10^{-8}$	.99	372.08
155.1	$1.6004x10^{-3}$	$6.2684x10^{-3}$	$-4.1250x10^{-7}$	$5.5638x10^{-8}$	.98	181.49
155.2	$-1.9330x10^{-2}$	$5.8714x10^{-3}$	$1.0066x10^{-6}$	$4.0539x10^{-8}$	.97	111.79
165.0	$-6.3007x10^{-3}$	$5.0926x10^{-3}$	$-2.4556x10^{-6}$	$-1.0821x10^{-8}$	.99	171.11
167.0	$3.2794x10^{-2}$	$5.7384x10^{-3}$	$-7.2646x10^{-6}$	$7.3429z10^{-9}$	.99	161.60
183.1	$-2.1641x10^{-2}$	$5.5873x10^{-3}$	$-4.0288x10^{-6}$	$4.1650x10^{-8}$	.98	189.14
183.2	$-1.0720x10^{-1}$	$5.6674x10^{-3}$	$-1.1032x10^{-6}$	$3.0873x10^{-8}$	.97	95.67
242.1	$-1.3838x10^{-2}$	$7.2949x10^{-3}$	$6.2518x10^{-7}$	$4.3745x10^{-8}$	.97	94.64
242.2	$-3.4428x10^{-2}$	$6.6084x10^{-3}$	$2.9994x10^{-8}$	$1.4158x10^{-8}$	.98	165.65
244.1	$3.2085x10^{-2}$	$6.7706x10^{-3}$	$2.1936x10^{-6}$	$5.1349x10^{-8}$	.97	114.99
244.2	$-4.4324x10^{-2}$	$8.3119x10^{-3}$	$-8.2027x10^{-6}$	$5.0713x10^{-8}$	.96	100.33
248.1	$-5.2233x10^{-2}$	$6.5531x10^{-3}$	$-4.6105x10^{-6}$	$6.2054x10^{-8}$	.99	249.65
248.2	$-7.3713x10^{-2}$	$6.5912x10^{-3}$	$-8.2052x10^{-6}$	$7.8889x10^{-8}$	.99	287.96
250.1	$-2.7480x10^{-2}$	$5.6118x10^{-3}$	$-3.1133x10^{-6}$	$6.9130x10^{-9}$	.98	156.06
250.2	$1.9801x10^{-2}$	$6.1156x10^{-3}$	$-3.0087x10^{-6}$	$3.6106x10^{-8}$	.99	407.03
251.1	$3.5029x10^{-4}$	$6.5451x10^{-3}$	$-9.5988x10^{-7}$	$7.3408x10^{-8}$	.99	472.48
251.2	$1.1502x10^{-2}$	$5.2853x10^{-3}$	$-1.2135x10^{-6}$	$1.3408x10^{-8}$	.99	603.97
290.1	$-2.6194x10^{-2}$	$6.7802x10^{-3}$	$8.0965x10^{-6}$	$8.9535x10^{-8}$	.97	100.18
290.2	$1.1975x10^{-2}$	$6.8452x10^{-3}$	$-4.7400x10^{-6}$	$6.0953x10^{-8}$	.97	94.58
317.1	$-2.3860x10^{-1}$	$5.9021x10^{-3}$	$-8.9515x10^{-6}$	$5.1670x10^{-8}$	.95	64.83
317.2	$-1.0157x10^{-2}$	$5.9245x10^{-3}$	$-2.9011x10^{-6}$	$1.4821x10^{-8}$	.98	185.96
Mean	$-1.323x10^{-2}$	$6.14x10^{-3}$	$-2.0096x10^{-6}$	$4.2370x10^{-8}$		

*Due to fourth degree polynomial equation.

Appendix 2A

Coefficients of the Highest Power of X Derived from Stepwise Polynomial Equations Maxillary Arch

DIZYGOTIC TWINS

Subject	B_{1_a}	B_{2_b}	B_{3_c}	B_{4_d}	Overall* R^2	Overall* F
89.1	$-2.9913x10^{-2}$	$4.9476x10^{-3}$	$2.6764x10^{-7}$	$3.0836x10^{-8}$	.98	155.39
89.2	$5.7664x10^{-3}$	$5.2080x10^{-3}$	$-8.8313x10^{-7}$	$3.2791x10^{-8}$	.98	182.12
123.1	$2.0583x10^{-2}$	$5.6751x10^{-3}$	$1.1999x10^{-6}$	$5.7354x10^{-8}$	.94	46.16
123.2	$7.5038x10^{-3}$	$6.2832x10^{-3}$	$-1.1922x10^{-7}$	$5.3178x10^{-8}$	.97	120.19
174.1	$8.3523x10^{-2}$	$5.4402x10^{-3}$	$1.2363x10^{-6}$	$4.2525x10^{-8}$	.98	194.94
174.2	$-1.1112x10^{-2}$	$5.5925x10^{-3}$	$-2.5540x10^{-6}$	$6.1548x10^{-8}$	.98	166.92
241.1	$4.3128x10^{-2}$	$5.0436x10^{-3}$	$6.7527x10^{-7}$	$3.5667x10^{-8}$	.99	151.52
241.2	$-7.5226x10^{-2}$	$6.2455x10^{-3}$	$-4.3275x10^{-6}$	$4.4926x10^{-8}$	1.00	691.43
247.0	$1.3469x10^{-2}$	$6.9213x10^{-3}$	$-1.4462x10^{-6}$	$8.9944x10^{-8}$	.98	140.73
248.0	$-3.0566x10^{-2}$	$5.6090x10^{-3}$	$2.7044x10^{-7}$	$2.4929x10^{-8}$	.99	352.97
276.0	$3.4074x10^{-2}$	$4.8751x10^{-3}$	$1.8793x10^{-6}$	$5.1760x10^{-10}$	.91	24.17
277.0	$-6.4459x10^{-2}$	$5.9266x10^{-3}$	$-4.6165x10^{-6}$	$-2.0175x10^{-9}$	.98	177.99
304.0	$-2.2098x10^{-2}$	$5.7507x10^{-3}$	$-1.3129x10^{-6}$	$5.8081x10^{-8}$	.97	123.82
305.0	$5.4803x10^{-3}$	$4.8682x10^{-3}$	$-1.0707x10^{-6}$	$3.2548x10^{-8}$	.98	211.62
333.1	$-2.2185x10^{-3}$	$5.7114x10^{-3}$	$-4.3098x10^{-6}$	$3.8255x10^{-8}$	.98	182.65
333.2	$7.6418x10^{-3}$	$6.3995x10^{-3}$	$9.6650x10^{-7}$	$4.9229x10^{-8}$	.97	120.89
903.2	$-5.9809x10^{-2}$	$4.9994x10^{-3}$	$-2.6375x10^{-6}$	$3.7268x10^{-8}$	1.00	960.63
903.3	$-2.8316x10^{-2}$	$5.8795x10^{-3}$	$3.0775x10^{-6}$	$7.2152x10^{-8}$	.97	90.05
906.2	$8.9917x10^{-3}$	$5.0021x10^{-3}$	$-1.9130x10^{-7}$	$4.4721x10^{-8}$	.98	168.74
906.3	$-8.8815x10^{-2}$	$4.1704x10^{-3}$	$-1.9284x10^{-6}$	$4.1154x10^{-8}$	.99	305.64

Appendix 2A (continued)

Subject	B_{1_a}	B_{2_b}	B_{3_c}	B_{4_d}	R^2	F
907.2	$-2.7929x10^{-2}$	$5.3260x10^{-3}$	$-1.8370x10^{-6}$	$5.3264x10^{-8}$	.98	210.58
907.3	$-2.2210x10^{-2}$	$5.0184x10^{-3}$	$-1.9393x10^{-6}$	$1.9368x10^{-8}$	.99	474.31
908.1	$-3.1741x10^{-2}$	$4.6458x10^{-3}$	$7.6218x10^{-7}$	$7.3376x10^{-9}$	.97	112.97
908.2	$-4.3291x10^{-2}$	$5.4285x10^{-3}$	$-4.1710x10^{-6}$	$3.9720x10^{-8}$	.97	117.86
911.2	$-8.0304x10^{-2}$	$5.8271x10^{-3}$	$-4.1401x10^{-6}$	$1.5492x10^{-8}$	.99	258.00
911.3	$-1.7705x10^{-2}$	$6.2446x10^{-3}$	$-3.5109x10^{-7}$	$7.0931x10^{-8}$	.97	102.30
Mean	$-1.5x10^{-2}$	$5.501x10^{-3}$	$-1.058x10^{-6}$	$4.045x10^{-8}$		

* Due to fourth degree polynomial equation

Appendix 2B

Coefficients of the Highest Power of X Derived from Stepwise Polynomial Equations
Mandibular Arch

DIZYGOTIC TWINS

Subject	B_{1_a}	B_{2_b}	B_{3_c}	B_{4_d}	Overall* R^2	Overall* F
89.1	$-2.9913x10^{-2}$	$4.9476x10^{-3}$	$2.6764x10^{-7}$	$3.0836x10^{-8}$	.98	155.39
89.2	$5.7664x10^{-3}$	$5.2080x10^{-3}$	$-8.8313x10^{-7}$	$3.2791x10^{-8}$	.98	182.12
123.1	$2.0583x10^{-2}$	$5.6751x10^{-3}$	$1.1999x10^{-6}$	$5.7354x10^{-8}$	.94	46.16
123.2	$7.5038x10^{-3}$	$6.2832x10^{-3}$	$-1.1922x10^{-7}$	$5.3178x10^{-8}$	.97	120.19
174.1	$8.3623x10^{-2}$	$5.4402x10^{-3}$	$1.2363x10^{-6}$	$4.2525x10^{-8}$	.98	194.94
174.2	$-1.1112x10^{-2}$	$5.5925x10^{-3}$	$-2.5540x10^{-6}$	$6.1548x10^{-8}$	.98	166.92
241.1	$4.3128x10^{-2}$	$5.0436x10^{-3}$	$6.7527x10^{-7}$	$3.5667x10^{-8}$	.99	151.52
241.2	$-7.5226x10^{-2}$	$6.2455x10^{-3}$	$-4.3275x10^{-6}$	$4.4926x10^{-8}$	1.00	691.43
247.0	$1.3469x10^{-2}$	$6.9213x10^{-3}$	$-1.4462x10^{-6}$	$8.9944x10^{-8}$	.98	140.73
248.0	$-3.0566x10^{-2}$	$5.6090x10^{-3}$	$2.7044x10^{-7}$	$2.4929x10^{-8}$	.99	352.97
276.0	$3.4074x10^{-2}$	$4.8751x10^{-3}$	$1.8793x10^{-6}$	$5.1760x10^{-10}$	.91	24.17
277.0	$-6.4459x10^{-2}$	$5.9266x10^{-3}$	$-4.6165x10^{-6}$	$-2.0175x10^{-9}$	.98	177.99
304.0	$-2.2098x10^{-2}$	$5.7507x10^{-3}$	$-1.3129x10^{-9}$	$5.8081x10^{-8}$	.97	123.82
305.0	$5.4803x10^{-3}$	$4.8682x10^{-3}$	$-1.0707x10^{-6}$	$3.2548x10^{-8}$	.98	211.62
333.1	$-2.2185x10^{-3}$	$5.7114x10^{-3}$	$-4.3098x10^{-6}$	$3.8255x10^{-8}$	.98	182.65
333.2	$7.6418x10^{-3}$	$6.3995x10^{-3}$	$9.6650x10^{-7}$	$4.9229x10^{-8}$	.97	120.89
903.2	$-5.9809x10^{-2}$	$4.9994x10^{-3}$	$-2.6375x10^{-6}$	$3.7268x10^{-8}$	1.00	960.63
903.3	$-2.8316x10^{-2}$	$5.8795x10^{-3}$	$3.0775x10^{-6}$	$7.2152x10^{-8}$	.97	90.05
906.2	$8.9917x10^{-3}$	$5.0021x10^{-3}$	$-1.9130x10^{-7}$	$4.4721x10^{-8}$	.98	168.74
906.3	$-8.8815x10^{-2}$	$4.1704x10^{-3}$	$-1.9284x10^{-6}$	$4.1154x10^{-8}$	.99	305.64

Appendix 2B (continued)

Subject	B_{1} a	B_{2} b	B_{3} c	B_{4} d	R^2	F
907.2	-2.7929×10^{-2}	5.0021×10^{-3}	-1.8370×10^{-6}	5.3264×10^{-8}	.98	210.58
907.3	-2.2210×10^{-2}	5.0184×10^{-3}	-1.9393×10^{-6}	1.9368×10^{-8}	.99	474.31
908.1	-3.1741×10^{-2}	4.6458×10^{-3}	7.6218×10^{-7}	7.3376×10^{-9}	.97	112.97
908.2	-4.3291×10^{-2}	5.4285×10^{-3}	-4.1710×10^{-6}	3.9720×10^{-8}	.97	117.86
911.2	-8.0304×10^{-2}	5.8271×10^{-3}	-4.1401×10^{-6}	1.5492×10^{-8}	.99	258.00
911.3	-1.7705×10^{-2}	6.2446×10^{-3}	-3.5109×10^{-7}	7.0931×10^{-8}	.97	102.30
Mean	-1.5×10^{-2}	5.501×10^{-3}	-1.058×10^{-6}	4.045×10^{-8}		

* Due to fourth degree polynomial equation

Appendix 3A

Coefficients of the Highest Power of X Derived from Stepwise Polynomial Equations
Maxillary Arch

SIBLING PAIRS

Subject	B_{1_a}	B_{2_b}	B_{3_c}	B_{4_d}	Overall* R^2	Overall* F
1.0	$4.7506x10^{-2}$	$4.0979x10^{-3}$	$1.2647x10^{-6}$	$2.4290x10^{-8}$	.99	518.93
2.0	$4.2804x10^{-2}$	$5.4525x10^{-3}$	$4.8017x10^{-6}$	$6.9377x10^{-8}$	.98	196.06
5.0	$1.4961x10^{-2}$	$5.9913x10^{-3}$	$1.7939x10^{-6}$	$4.9666x10^{-8}$	.96	71.90
5.1	$2.2272x10^{-2}$	$5.0208x10^{-3}$	$2.5449x10^{-6}$	$1.7859x10^{-8}$	.99	277.05
21.0	$2.9240x10^{-3}$	$4.5194x10^{-3}$	$-6.9814x10^{-7}$	$2.7106x10^{-8}$	1.00	1047.70
22.0	$2.4004x10^{-3}$	$4.5733x10^{-3}$	$1.1384x10^{-6}$	$2.5625x10^{-8}$	.99	269.21
32.0	$-6.7746x10^{-4}$	$4.7952x10^{-3}$	$1.4119x10^{-6}$	$2.3486x10^{-8}$	.96	82.18
33.0	$-4.3043x10^{-2}$	$4.9984x10^{-3}$	$-4.7156x10^{-6}$	$2.4530x10^{-8}$	.97	93.93
47.0	$7.3043x10^{-2}$	$4.8846x10^{-3}$	$1.2346x10^{-6}$	$2.3015x10^{-8}$	.99	264.67
48.0	$3.5038x10^{-2}$	$5.5004x10^{-3}$	$1.0796x10^{-6}$	$4.6275x10^{-8}$	.99	580.23
52.0	$-1.5661x10^{-2}$	$6.5132x10^{-3}$	$-2.4814x10^{-6}$	$7.7321x10^{-8}$	.99	302.99
53.0	$-6.1077x10^{-3}$	$5.9547x10^{-3}$	$3.9979x10^{-7}$	$3.7705x10^{-8}$	.98	191.80
67.0	$1.8955x10^{-2}$	$6.1294x10^{-3}$	$-4.2538x10^{-7}$	$3.6883x10^{-8}$	.99	110.17
68.0	$-6.6210x10^{-2}$	$5.4295x10^{-3}$	$-1.9607x10^{-6}$	$1.5675x10^{-8}$	.99	272.09
82.0	$-2.3593x10^{-3}$	$5.8823x10^{-3}$	$-7.9151x10^{-8}$	$1.8338x10^{-8}$	1.00	752.96
83.0	$1.3510x10^{-2}$	$6.6287x10^{-3}$	$2.4180x10^{-6}$	$1.5980x10^{-8}$	.99	309.68
87.0	$1.2212x10^{-2}$	$5.7435x10^{-3}$	$2.0486x10^{-6}$	$4.0541x10^{-8}$	.99	535.05
89.0	$-1.8160x10^{-2}$	$4.8003x10^{-3}$	$-2.2966x10^{-7}$	$1.9006x10^{-8}$	.99	304.32
107.0	$-2.6538x10^{-2}$	$5.3510x10^{-3}$	$1.1151x10^{-7}$	$2.7702x10^{-8}$	.99	532.07
108.0	$-2.7754x10^{-2}$	$5.3180x10^{-3}$	$-2.8879x10^{-6}$	$2.5194x10^{-8}$	.99	326.74
116.0	$4.0474x10^{-3}$	$6.0007x10^{-3}$	$3.0932x10^{-6}$	$2.5135x10^{-8}$	.98	131.85
117.0	$2.1248x10^{-2}$	$4.6205x10^{-3}$	$1.1552x10^{-6}$	$2.5000x10^{-8}$	.99	391.69

Appendix 3A (continued)

Subject	B_{1_a}	B_{2_b}	B_{3_c}	B_{4_d}	R^2	F
118.0	$4.1156x10^{-3}$	$5.4484x10^{-3}$	$-1.2889x10^{-7}$	$2.0610x10^{-8}$	.99	324.91
119.0	$2.1791x10^{-2}$	$5.7136x10^{-3}$	$-1.4052x10^{-6}$	$3.6096x10^{-8}$	.99	393.11
140.0	$3.8983x10^{-2}$	$4.4131x10^{-3}$	$3.6911x10^{-6}$	$6.4856x10^{-8}$	.96	84.18
141.0	$-7.6755x10^{-5}$	$6.5818x10^{-3}$	$1.3799x10^{-6}$	$5.5617x10^{-8}$	.89	18.30
151.0	$-1.0100x10^{-2}$	$4.9368x10^{-3}$	$1.6319x10^{-7}$	$1.0557x10^{-8}$	1.00	615.62
152.0	$-1.5172x10^{-2}$	$3.8839x10^{-3}$	$-6.7441x10^{-8}$	$1.5828x10^{-8}$	1.00	1227.14
157.0	$1.0394x10^{-2}$	$4.9947x10^{-3}$	$8.9487x10^{-9}$	$1.6385x10^{-9}$	.98	176.65
158.0	$9.0998x10^{-3}$	$5.1477x10^{-3}$	$-4.7454x10^{-7}$	$2.4687x10^{-8}$	.99	315.25
173.0	$3.9412x10^{-2}$	$5.0079x10^{-3}$	$2.3374x10^{-6}$	$3.8168x10^{-8}$	.98	150.21
174.0	$-2.0156x10^{-2}$	$6.0868x10^{-3}$	$6.6269x10^{-7}$	$3.1426x10^{-9}$	.98	167.01
192.0	$3.5101x10^{-4}$	$4.6569x10^{-3}$	$-1.6514x10^{-6}$	$1.9223x10^{-9}$	.98	148.87
195.0	$-2.7858x10^{-2}$	$5.5793x10^{-3}$	$-1.5807x10^{-6}$	$2.6172x10^{-8}$	.99	489.06
199.0	$-4.7307x10^{-2}$	$6.0330x10^{-3}$	$-1.7522x10^{-6}$	$1.3545x10^{-8}$	.99	268.02
200.0	$1.0662x10^{-2}$	$4.8990x10^{-3}$	$-1.0754x10^{-6}$	$3.5790x10^{-8}$	.99	464.83
213.0	$-5.0373x10^{-2}$	$4.3881x10^{-3}$	$-2.9551x10^{-6}$	$1.9840x10^{-8}$	.99	373.56
214.0	$-4.5675x10^{-2}$	$4.9887x10^{-3}$	$-3.6333x10^{-6}$	$1.1648x10^{-8}$	.99	629.45
222.0	$4.2715x10^{-2}$	$4.1771x10^{-3}$	$3.0572x10^{-6}$	$2.9067x10^{-8}$	1.00	781.47
223.0	$1.4123x10^{-2}$	$4.7381x10^{-3}$	$1.0083x10^{-6}$	$4.8655x10^{-8}$	.99	425.58
224.0	$1.2071x10^{-2}$	$6.2096x10^{-3}$	$-4.5818x10^{-6}$	$2.8460x10^{-8}$	.99	234.30
225.0	$2.6551x10^{-2}$	$6.2050x10^{-3}$	$2.7056x10^{-6}$	$1.0600x10^{-8}$	1.00	676.79
Mean	$1.25x10^{-3}$	$5.319x10^{-3}$	$0.160x10^{-6}$	$2.907x10^{-8}$		

* Due to fourth degree polynomial equation

Appendix 3B

Coefficients of the Highest Power of X Derived from Stepwise Polynomial Equations
Mandibular Arch

SIBLING PAIRS

Subject	B_{1_a}	B_{2_b}	B_{3_c}	B_{4_d}	Overall* R^2	Overall* F
1.0	2.8557×10^{-2}	4.5293×10^{-3}	2.8270×10^{-6}	3.3265×10^{-8}	.99	487.13
2.0	4.8682×10^{-2}	5.5023×10^{-3}	3.4675×10^{-6}	4.0769×10^{-8}	.99	215.75
5.0	-1.7812×10^{-2}	6.9649×10^{-3}	4.3422×10^{-6}	7.0563×10^{-8}	.97	115.34
5.1	2.8571×10^{-2}	5.9430×10^{-3}	2.6428×10^{-6}	3.3734×10^{-8}	.99	203.54
21.0	-1.9831×10^{-2}	5.1230×10^{-2}	-1.4801×10^{-6}	3.0309×10^{-8}	1.00	844.07
22.0	7.7651×10^{-3}	5.1968×10^{-3}	-2.0938×10^{-6}	3.4350×10^{-8}	.99	400.97
32.0	1.4507×10^{-2}	4.3188×10^{-3}	-8.1203×10^{-7}	2.6356×10^{-8}	.97	118.97
33.0	-5.1300×10^{-2}	5.2226×10^{-3}	-4.4509×10^{-6}	1.8036×10^{-8}	.99	240.41
47.0	1.3873×10^{-2}	5.4729×10^{-3}	6.8018×10^{-7}	2.3951×10^{-8}	.99	458.68
48.0	9.1176×10^{-3}	6.1784×10^{-3}	1.2273×10^{-6}	8.7521×10^{-8}	.98	210.50
52.0	-3.1967×10^{-2}	6.7323×10^{-3}	-1.4434×10^{-6}	5.7195×10^{-8}	1.00	2695.75
53.0	-2.8018×10^{-2}	6.6082×10^{-3}	-1.6041×10^{-6}	3.7857×10^{-8}	.99	460.69
67.0	-2.2520×10^{-3}	6.7157×10^{-3}	1.0165×10^{-6}	3.9088×10^{-8}	.99	452.91
68.0	-1.4729×10^{-3}	5.8708×10^{-3}	-3.8492×10^{-6}	3.9253×10^{-8}	1.00	1798.10
82.0	-1.2049×10^{-2}	6.6545×10^{-3}	-5.7732×10^{-7}	2.9980×10^{-8}	.99	333.70
83.0	4.5119×10^{-3}	7.6271×10^{-3}	2.6596×10^{-7}	5.9813×10^{-9}	.99	463.13
87.0	-1.6612×10^{-2}	6.1335×10^{-3}	3.4481×10^{-6}	6.8159×10^{-8}	.99	390.49
89.0	-1.0939×10^{-2}	5.7171×10^{-3}	1.3308×10^{-6}	3.3508×10^{-8}	1.00	1392.09
107.0	2.9474×10^{-2}	5.4252×10^{-3}	3.4472×10^{-6}	2.3091×10^{-8}	.99	520.74
108.0	-2.7635×10^{-2}	6.1588×10^{-3}	-3.3037×10^{-8}	2.5360×10^{-8}	.99	436.10
116.0	2.9503×10^{-3}	6.0600×10^{-3}	3.3383×10^{-6}	2.7923×10^{-8}	.98	161.01
117.0	2.5729×10^{-2}	4.8357×10^{-3}	6.6528×10^{-7}	2.8342×10^{-8}	.99	585.19

Appendix 3B (continued)

Subject	B_{1_a}	B_{2_b}	B_{3_c}	B_{4_d}	R^2	F
118.0	$7.0027x10^{-3}$	$6.4034x10^{-3}$	$3.1156x10^{-6}$	$3.1080x10^{-8}$	.98	211.19
119.0	$2.8594x10^{-2}$	$7.0078x10^{-3}$	$-4.6257x10^{-6}$	$4.1397x10^{-8}$	.96	96.44
140.0	$4.8437x10^{-2}$	$6.3179x10^{-3}$	$1.4145x10^{-6}$	$8.5077x10^{-8}$	.98	135.11
141.0	$-4.8748x10^{-3}$	$6.8938x10^{-3}$	$-2.2831x10^{-6}$	$1.6487x10^{-7}$	.98	19.05
151.0	$-3.1542x10^{-3}$	$5.9467x10^{-3}$	$2.0633x10^{-6}$	$2.6995x10^{-8}$	.99	488.66
152.0	$-1.2841x10^{-2}$	$4.9329x10^{-3}$	$4.9843x10^{-8}$	$2.9387x10^{-8}$	1.00	686.96
157.0	$8.0491x10^{-2}$	$6.0725x10^{-3}$	$-7.5790x10^{-7}$	$3.3360x10^{-9}$	1.00	622.32
158.0	$1.1540x10^{-2}$	$5.8639x10^{-3}$	$-5.2967x10^{-7}$	$3.8668x10^{-8}$	.98	211.13
173.0	$3.0867x10^{-2}$	$5.0342x10^{-3}$	$3.2408x10^{-6}$	$1.3122x10^{-8}$	.99	369.86
174.0	$-1.7888x10^{-7}$	$7.5497x10^{-3}$	$1.3493x10^{-6}$	$1.4677x10^{-8}$	.98	183.16
192.0	$-2.5221x10^{-2}$	$5.1042x10^{-3}$	$1.7542x10^{-6}$	$2.1675x10^{-8}$	.99	278.58
195.0	$7.2852x10^{-3}$	$5.9253x10^{-3}$	$-2.4879x10^{-6}$	$1.5703x10^{-8}$	1.00	696.13
199.0	$-1.3550x10^{-2}$	$6.9026x10^{-3}$	$-3.7714x10^{-6}$	$2.3189x10^{-11}$	.98	173.10
200.0	$1.0722x10^{-2}$	$5.6043x10^{-3}$	$7.6313x10^{-7}$	$3.7637x10^{-8}$	.99	489.10
213.0	$-2.1859x10^{-2}$	$5.0086x10^{-3}$	$-3.5260x10^{-6}$	$2.4291x10^{-8}$	.00	340.04
214.0	$-2.9678x10^{-2}$	$5.6293x10^{-3}$	$-6.5010x10^{-6}$	$1.8543x10^{-8}$	.99	286.09
222.0	$6.1504x10^{-2}$	$4.8287x10^{-3}$	$3.6654x10^{-6}$	$3.1625x10^{-8}$	.99	276.12
223.0	$-2.2467x10^{-2}$	$6.2362d10^{-3}$	$-9.7830x10^{-7}$	$6.0858x10^{-8}$	.99	331.05
224.0	$1.9894x10^{-2}$	$7.1103x10^{-3}$	$-5.6775x10^{-6}$	$3.2206x10^{-8}$	.96	69.24
225.0	$-2.5200x10^{-3}$	$6.7448x10^{-3}$	$4.5337x10^{-6}$	$1.8861x10^{-8}$	.99	280.06
Mean	$3.18x10^{-3}$	$5.95x10^{-3}$	$-0.722x10^{-7}$	$3.6302x10^{-8}$		

* Due to fourth degree polynomial equation

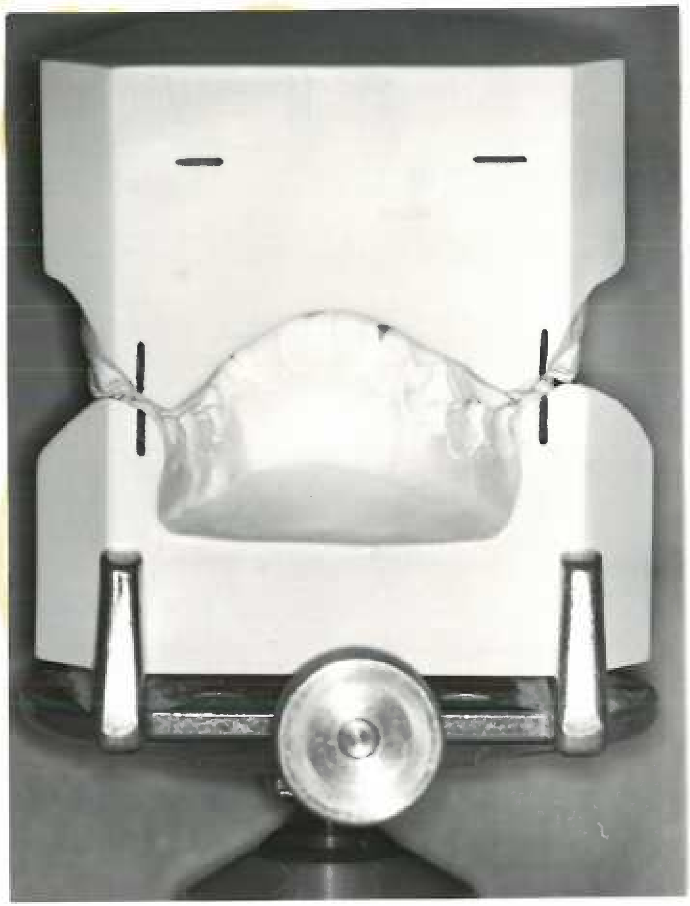

Figure 1: View of the heels of the upper and lower study casts showing two horizontal marks on the upper cast. Using these marks as a guide, two vertical marks were scribed on both upper and lower casts to be projected onto the occlusal surface of each arch.

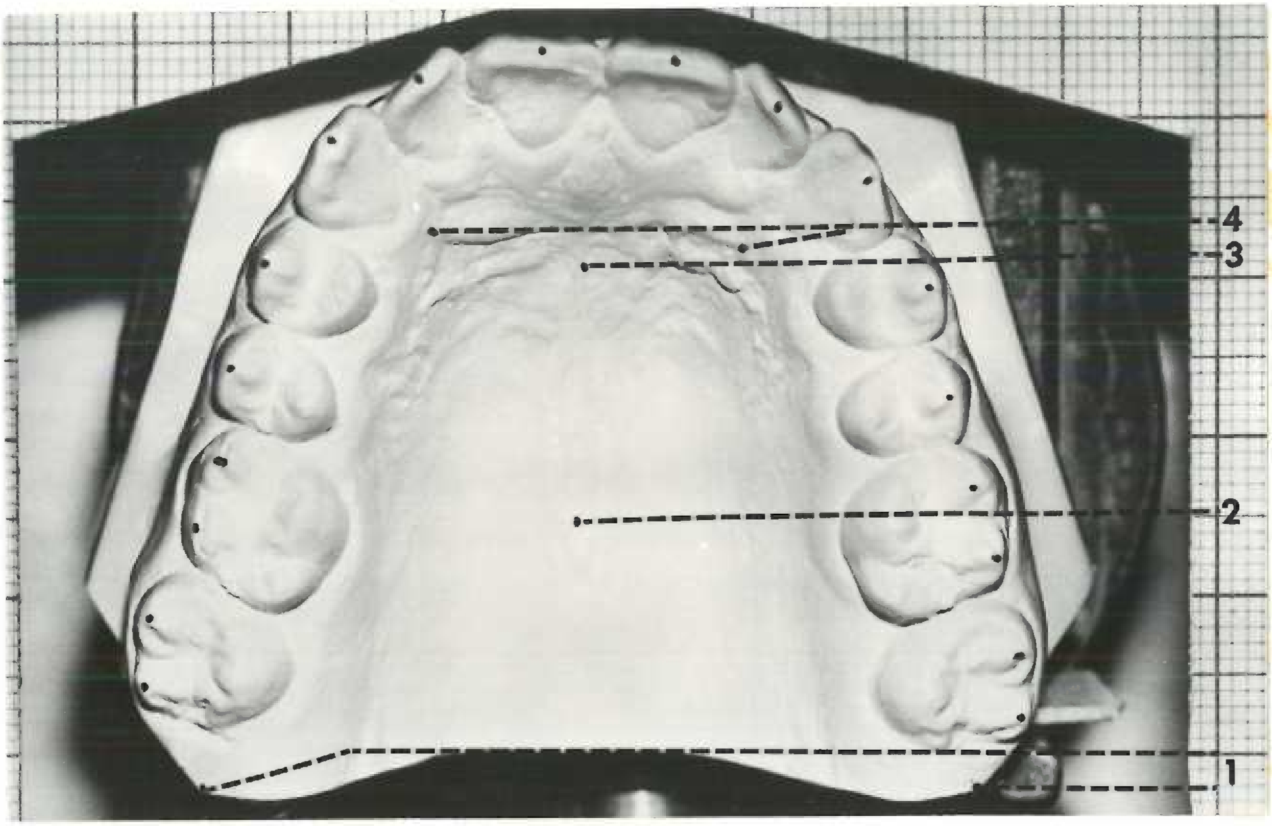

Figure 2: Occlusal view of upper dental cast with tooth cusps and incisal edges marked as well as:

#1 - transferred heel marks

#2 - most dorsal point on midpalatal raphe

#3 - most ventral point on midpalatal raphe

#4 - lateral termination of the most anterior rugae

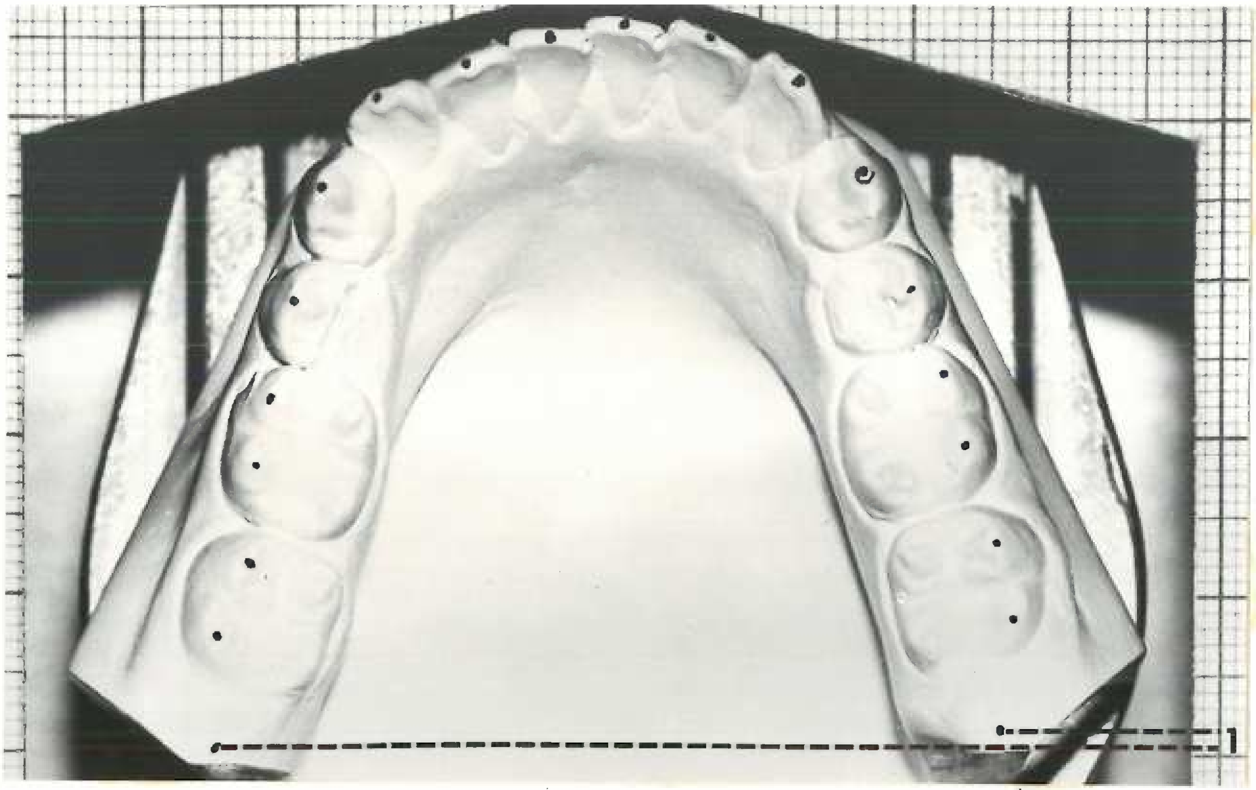

Figure 2 (continued): Occlusal view of lower dental cast with tooth cusps and incisal edges marked as well as:

#1 - transferred heel marks

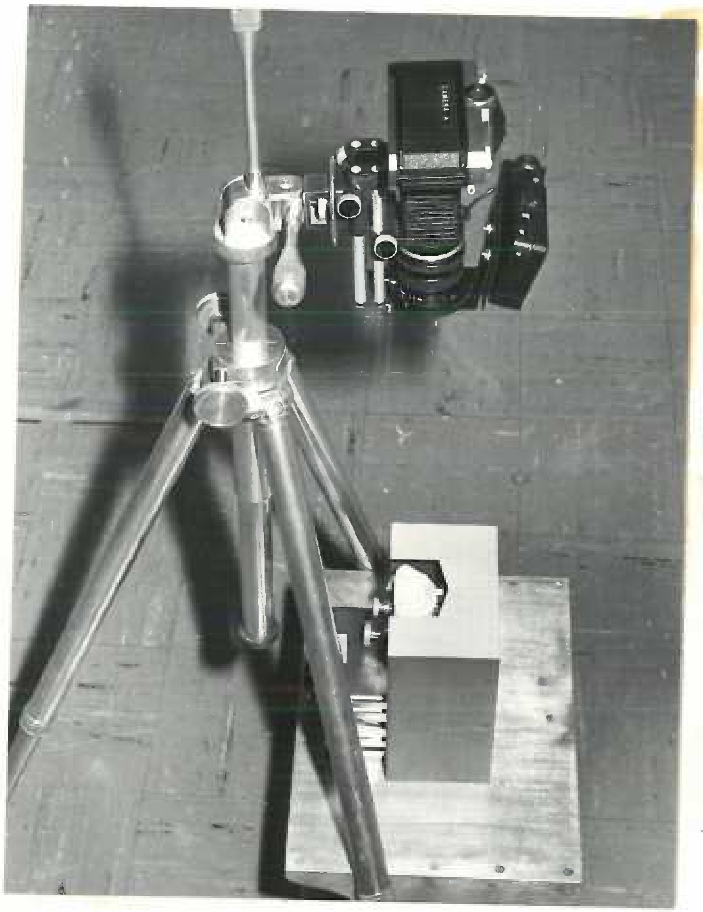

Figure 3: The camera set up. A Nikkormat camera with bellows and 100 mm short mount lens, mounted on a tripod. The dental cast on a surveyor table, with its occlusal surface flush with the top of the orientation table.

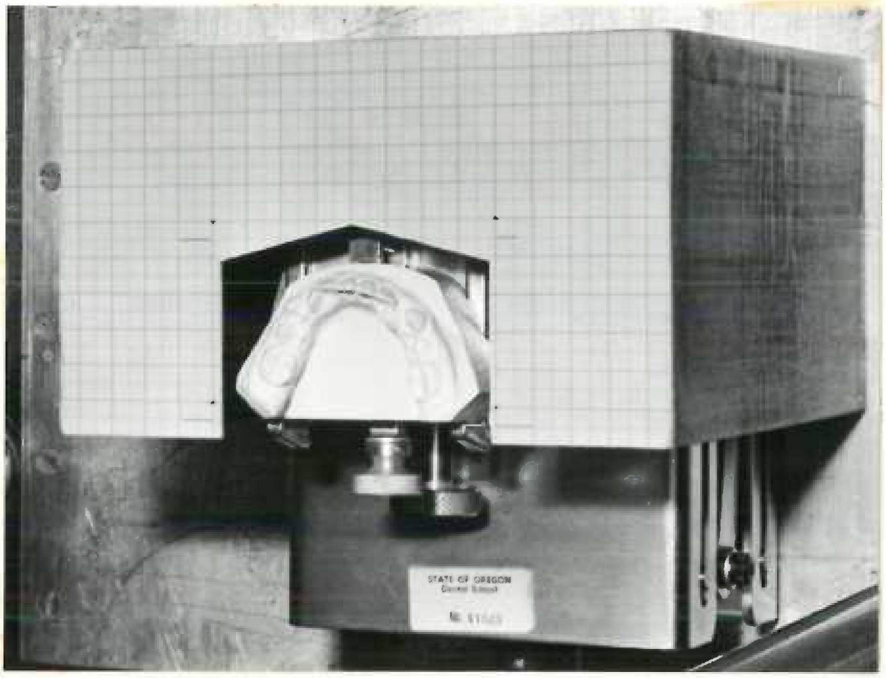

Figure 4: View from the top of the orientation table with a dental cast in place. Four fiducial marks are visible on the graph paper surface of the orientation table.