Dental Arch Stability

by

Shufang A. Lai

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To my late grandfather

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Dental Arch Stability

Introduction

Researchers have studied arch form and arch stability for more than a century. There are many arch forms, U-shaped, tapered , ovoid, square, circular, parabolic, and hyperbolic.[1-7] However, the 'ideal' arch form is still unclear.

Various hypotheses exist regarding the determinants of arch form. These hypotheses include facial pattern [4,6,8], genetic [4,9], substance of teeth [10,11], intra-oral force [4,14,15], osseous base [16], growth [16-18], axial inclination of teeth [18], and neutral force [12,13] (the catenary as the curve assumed by a fine chain of many links suspended by its ends and allowed to hang freely [12]). Intraorally teeth are tied together by transeptal fibers and the arch form is maintained by equal amounts of alveolar growth[13].

In a brief review of the literature, arch form does not appear to be stable over time and is subject to many chages, especially during the mixed dentition. Speck [19] studied the developmental changes in the human dental arch and concluded that the arch form becomes flatter and wider in the anterior and wider in the posterior during the transition from the deciduous to permanent dention. Barrow and White [5] found increases in intercuspid width of 4 mm in the maxilla and 3 mm in the mandible between the ages of 5 and 9.The intercuspid width then decreased 0.5 to 1.5 mm after the age of 14. The molar increased in width 1.8 mm in the maxilla and 1.2 mm in the mandible between the ages of 7 and 11.

Afterward, intermolar width decreased, on average, 0.4 mm in the maxilla and 0.7 mm in the mandible between the ages of 11 and 15. Howes[20] stated that the basal arch from the right to left mandibular first molar changed little after age 5 but the coronal arch could enlarge during the mixed dentition stage. Richardson and Brodie [21] suggested that both the size and shape of a dental arch changes maximally during the age span of 5 to 7 and 11 to 13. This change corresponds to major phases of tooth eruption. DeKock [22] found no significant changes in molar arch width in females after age 12 and slight increases of 0.6 mm to 1.0 mm between the ages of 12 and 15 in males. Knott [23] studied bicanine dimension and felt that this width is stable after the eruption of the permanent dentition.

The stability of orthdontic arch expansion is questionable [24-26]. Shapiro [25] and Gardner and Chaconas[26] found that the mandibular intercanine width showed a strong tendency to return to the pretreatment dimension in all classifications of malocclusions, except Class II div 2. Johnson[18] suggested that lower arch crowding after orthodontic treatment may be due to multiple factors. These factors include expanded cuspids, protrusive and labially inclined mandibular incisors and late growth. Finally, Water [27] and Riedel [28] suggested that general arch form should be a consideration in preventing relapse.

Mechanically, an arch is typically a structural member which spans an opening and serves as a support. It is a particular arrangement of blocks of material put together along a curved line in such a way that they resist a load. An arch resists a load by balancing certain thrusts and counter thrusts. The outward thrust on the arch requires that one use the arch with caution if the abutments are not of ample size and strength. In order that an arch maintain stability, the line of resistance must fall within the middle third of the arch. The line of resistance is dependent on the form and dimension of the arch as well as the points of loading on the arch.

The purpose of his study is to help one understand the concept of dental arch stability using the mechanical model of an arch as a guide.

Hypothesis

Besides the vertical component of occlusal force, two horizontal forces are created as a result of biting. One produces mesial drift to both upper and lower dentitions[29-47] (Appendix I), the other produces distal drift to the lower dentition as the mandible flexes (Appendix II)[47,50,69]. In the first condition (loading I), the upper and lower arches are locked together by good interdigitation. They move forward against the perioral musculature which is anchored by the buccinator and superior pharygeal constrictor muscles (Fig.1). As the mandible flexes (loading II), the lower dentition sustaines inward pressure from the upper arch which in turn sustaines outward pressure from the lower arch.

Lashar[54], in 1934, suggested his arch theory. He stated that there were three arches in the masticatory system, lower dental arch, upper dental arch and the arch which originates at the base of the mandible, on one side, passes up through the lower teeth to the upper teeth and over the roof of the mouth, then down on the opposite side. According to his mechanical understanding the only teeth in the true curve itself, were the six anterior units. He said the bicuspids and molars acted as buttresses to support the anterior arc. In addition, there were bony buttress (rami and the mandibular body in the lower arch and sphenoid bone and zygomatic ridge in the upper arch) acting to support the tooth buttresses. In the past various people believed Lashar' theory because of the similarity between the alignment of teeth in a dental arch and the alignment of voussoirs (see appendix 4 for definition) in a masonary arch. To Lashar, arch stability took on a mechanical model which he used to explain slipping of teeth, rotation of teeth and crushing of the arches from too much pressure. However, Lashar's theory is almost abandoned today due to the small contacting area between the teeth.

Can one adjust Lasher's theory to apply to the dental arch? In an oral environment, the design of a point contact between a tooth and its adjacent tooth is considered benificial. The less teeth interlock, the more freedom they allow for spacial adjustment. This adjustment is critical for the maintenance of equilibrium, however, this statement seems to contradict the principle of arch design, which states that an arch must be of sufficient stiffness and thickness to prevent it from buckling.

As part of my hypothesis one can assume the effective thickness of a dental arch to be about half the thickness of the labial-lingual dimension of a lower incisor. This thickness is affected by forces within the oral environment (mesial directed force or distal directed force). For a load which creats mesial drift to both the upper and lower dentition, the effective thickness of an arch is almost twice as thick as a load which creats distal drift to the lower dentition. Twice as much thickness is needed under conditions of mesial drift because both the upper and lower arches work as a unit, whereas, in distal drift only the lower arch is affected. For the arch (both upper and lower denture) to resist the mesial component of the occlusal force, the effective thickness of this arch is the distance from the upper contact point to the lower contact point. (See figure 4). For the arch (lower dentition) to resist the distal driving force from the upper denture, the effective thickness of the arch is about the half the thickness of the lower incisor. Due to the statement the arch can maintain its stability by rotating its unit (tooth) in the oral environment.

In a layer design, an arch and a cable juxtapose (Fig.1), thereby enhancing the endurance of the arch and eliminating the necessity of a stiffening truss for the cable. In this design an arch would be less pliable to overturning. Likewise, no failure of an arch would occur even if the point of application of force is located outside the middle third of the arch. Due to this layer design the teeth can maintain point contact and the arch maintains stability.

From a mechanical point of view, I suggest that there is a mesial directed force to both dentitions (loading 1) whenever they are brought into occlusal contact. Both dentitions hold together to sustain mesial migrating forces. The resistance to this mesial force comes from the roots of the teeth which are evenly distributed. If there is a necessity for controling an arch overturn, the muscular sling (perioral musculature, buccinator, and superior pharygeal constrictors) is called into function. Mechanically, the perioral musculature acts as a cable.(Fig. 1)

In contrast, when the mandible flexes as a result of the masticatory force (loading II), the resistance to this forward flex of the mandible would be centered in the upper arch. The lower arch sustains a number of evenly distributed forces which are parallel to the direction of mandibular flex but in an opposite direction (backward or/distal). If there is a necessity for controling arch overturn (line of resistance is outside the middle third of the arch), the maxillary arch would maintain the integrity of the lower arch (mandibular arch (arch) under compression and maxillary arch (cable) under tension). (See fig. 2 and fig. 3). Figure 2 demonstrates the relationship between dental arches and the muscular sling under loading 1 (mesial drift of both dentures). Figure 3 demonstrates the relationship between upper and lower arches under loading II (distal drift to the lower denture), tension to the upper arch and compression to the lower arch.

This research will study various malocclusions with different anterior teeth arrangements to prove or disprove the above hypothesis.

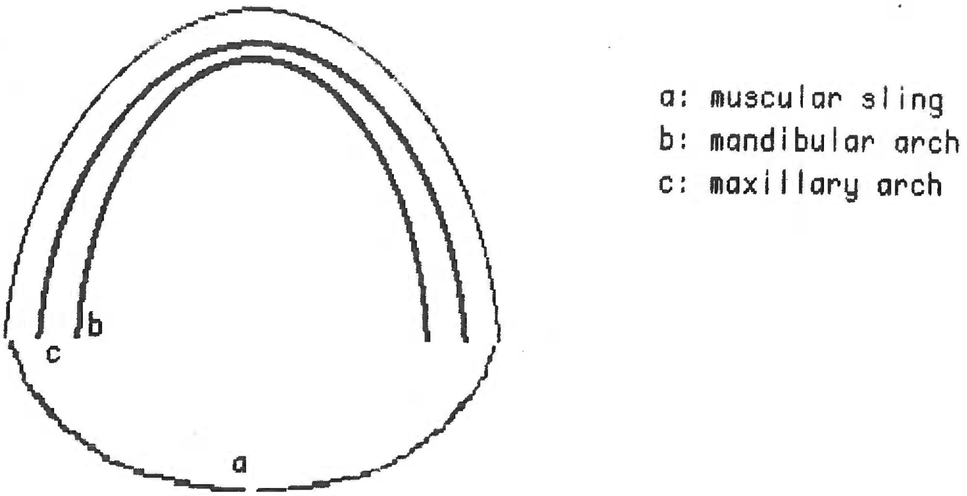


Figure 1. a layer design- muscular sling, mandibular arch and maxillary arch

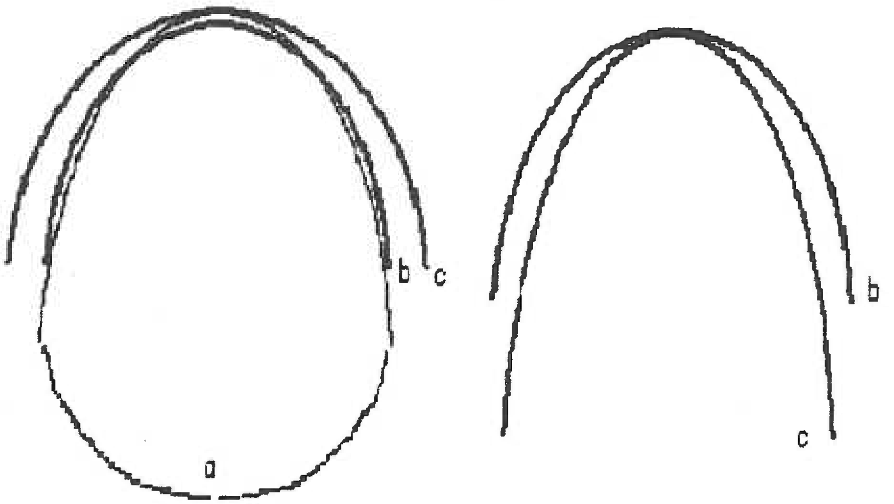


Fig. 2

Fig. 3

Figure. 2, the relationship between dentitions and perioral musculature when denture sustained mesial directed occlusal force . Figure 3, the relationship between upper dentition and lower dentition when the lower dentition sustained distal directed occlusal force from the upper dentition. (a: muscular sling, b: mandibular arch, c:maxillary arch)

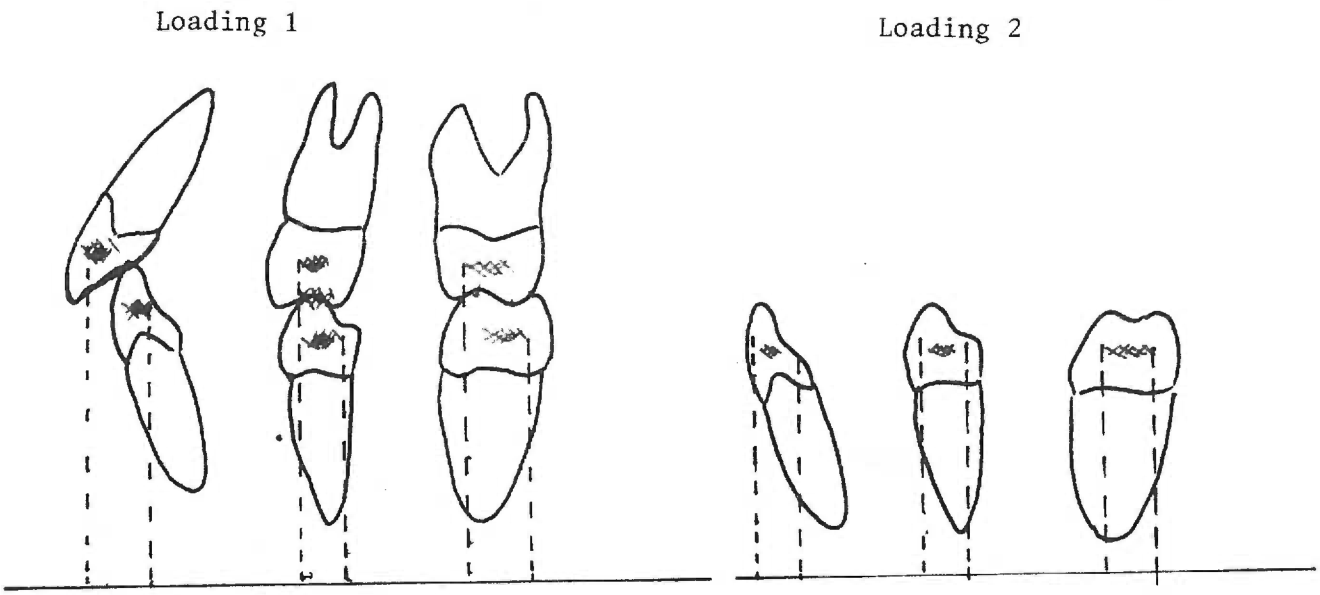


Figure 4, the effective thickness of an arch which sustained mesial and distal component of the occlusal force

Literature Review

Brodie[29], in 1934, stated that interproximal wear in the human dentition results in the movement of the posterior teeth forward and the front teeth backward. He believed the lip musculature restrained anterior tooth movement. Waldron [30], explained the mesial migration of molar and distal migration of premolar, canine and incisor teeth. He claimed that the resultant vector of occlusal force was in a forward direction, and is limited by the buccinator and the orbicularis oris muscles. In 1949, Dewell[31] said that the varying degrees of mesial inclination was characteristic for teeth. He felt the buccinator muscle crossed from buccal to lingual posterior to the last molar creating the pyterygomandibular raphe. This sling creates mesial pressure on the teeth. The masseter, temporal and internal pterygoid muscles also play a role in mesial migration.

Huckaba[32] believed that the buccinator muscle played a limiting role in mesial migration. He feels that as the crowns of the teeth migrate away from the basal bone in the process of eruption, their positions are governed by the environmental forces (buccinator) acting upon them. He supported Strang's[33] statement that the mandibular cuspid and first molar were the key in determining the alignment pattern for the remaining teeth, and that intercuspid and intermolar width could not be violated during treatment.

Begg[46] studied dentitional wear patterns of Australian Aboriginies and declared that the reduction of the mesiodistal dimention of the dentitions was created by both proximal wear and occlusal wear. No anterior component of occlusal force was mentioned in this study. In 1956, Newcomb[34] said that segments of teeth tended to tip toward and occupy the space created by interproximal wear. He believed that through proximal contact, the teeth are supported by their adjacent teeth and that without the proximal contact the anterior force of occlusion would be a harmful one.

Strang[35] presented a mechanical background for the theory of mesial drift. He felt the anterior component of force was a powerful forward propeling force associated with inclined plane action and arising as a result of the normal mesial axial inclination of the buccal teeth. This force caused the maxillary and mandibular teeth to meet one another in a series of angles. The apices of the angles were directed forward, so that the force component, emanating from such a relationship was disseminated toward the front of the mouth. This force was resisted by the great muscular bulwark that pressed against the labial surface of the incisors and delivered a backward restraining force that was transmitted to the canine, premolars and molars through the correct proximal contact points.

Most experimental studies on the mesial migration of teeth and anterior component of occlusal force took place after 1960.

Osborn[36] studied the interdental forces occurring between teeth of the same arch during clenching. He found an increased anterior force when the jaws were actively clenched as compared to the jaws in a relaxed state. Picton[37] studied the tilting movements of 60 teeth, ('cheek' teeth and upper central incisors), during biting. He found that the teeth tilted mesially under a load of 2 Kg for incisors and 5 Kg for 'cheek' teeth. Likewise, all the 'cheek' teeth anterior to the stressed tooth tilted mesially. This tilting effect spread to the central incisors. The presence of contacts between adjacent teeth was necessary for the transmission of force. In contrast, little movement was produced on posterior teeth when force was applied to the anterior teeth. According to Piction an anterior component of occlusal force was indeed present during biting.

Moss and Picton[38,39,45] in the late 1960, performed a series of monkey studies. After reducing the contact points between adjacent teeth, Moss and Picton[38] could not prove that occlusal force was a factor in causing mesial drift. On the contrary, they found that opposing molars retarded mesial tooth migration. In subsequent studies[39,45] they showed that the trans-septal fibre system played an important role in proximal drift. These authors felt that the cheeks and tongue did nothing as far as mesial drift was concerned.

Van Beek's studies seemed to support the occlusal factor in mesial drift. In 1977, Van Beek and Fidler[40] found that the rate of migration in first molars with no occlusal contact was slower than in those teeth that were left in occlusion, but whose intercuspation was modified. In a second study involving Macaca Irus, Van Beek[41] showed that when proximal drift was made possible by approximal and selective occlusal grinding, the second molars migrated mesially faster than the first. When the proximal contact between the first and second molars was untouched, the first molars tended to migrate faster. This study indicated that the drift potential might be transferred by proximal contact between teeth. Van Beek believed that this phenominum could not be explained by trans-septal fibre pull. Instead a horizontal vector of occlusal force was responsible for mesial drift. In a separate study, Picton[42] found no relationship between the angulation of roots and rate of proximal drift of cheek teeth, although intergroup variables were hard to rule out.

Other indirect evidence for the presence of proximal drift came from the cephalometric study of Richardson[43]. In a longitudinal study he found that the first molars moved mesially between the ages of 15 to 18. At the same time lower arch crowding increased and lower incisors tended to procline. He suggested that pressure from behind caused a forward movement of the lower first permanent molar, and subsequent lower arch crowding.

In a new approach. Steigman et al.[44] studied dentitions with generalized spacing. They found a tendency toward closure of buccal spaces and opening of anterior spaces. The most frequent site of space closure was located in the cuspid, first bicuspid region. New spaces appeared primarily at the mesial aspects of the upper and lower lateral incisors.

In terms of distal drift, Bjork's[47] implant study showed that forward rotation of the mandible displaced the paths of eruption of all the teeth in the mesial direction, thereby creating crowding in the anterior segment. In the 'backward rotator', the position of the lower incisors is functionally related to the upper arch. These teeth became retroclined in the mandible with a resultant decrease in alveolar prognathism. The lateral teeth are not guided distally in their eruption to the same extent as the anterior teeth, therefore, which leads to crowding in the anterior segment. Bjork's[48] longitudinal study, suggested that the average age change in alveolar prognathism of the maxilla was insignificant. In the mandible alveolar prognathism showed a tendency to diminish with age. Likewise, there existed a tendency for the intercisal angle to increase with age.

Lande[49] did a longitudinal study, from age 4 to 17, in 34 subjects. He found point B almost always changed less than gnathion in a horizontal direction. He concluded that alveolar bone growth did not keep pace with skeletal bone growth.

Other than growth, there was little evidence to support the theory of distal tooth drift. Witter[50] studied the migration of teeth in shortened dental arches. His sample included two groups, one less than 40 years old, the other greater than 40 years old. He stated that the remaining premolars in a shortened dental arch tend to migrate distally. The difference was smaller in the upper premolar region.

In summary, it appears that an anterior component of occlusal force exists, which create mesial tooth migration. In contrast, although there is some indication for distal drift, most researchers do not support this theory since there is no histological evidence to support physiological distal drift of teeth in humans.

In terms of ideal arch form, Hawley[6] recommendes that the arch be contructed upon an equilateral triangle. The six anterior teeth were thought to be arranged on the arc of a circle whose radius was determined by the combined width of the incisors and canine. The premolars and first permanent molars were arranged in a straight line with the second and third molars turning in toward the midline. William[10] was convinced that the anterior teeth should lay on an arc of a circle with its centre midway between the buccal grooves of the first molars. Izard[8] mentioned that there was a variety of arch forms, 75% ellipse, 20% parabolic and 5 % U shaped. In 1934, Lashar[54] proposed the structural theory of arch form. He believed that the posterior teeth buttresed the anterior teeth, and the cuspid could be compared to the springers (see appendix 4 for definition). He said the dental arches were buttressed and supported externally, internally, posteriorly and anteriorly. McConnail and Scher[12] thought the ideal curve of a dental arch would correspond to a catenary curve. Burdi[58] studied the dental arch shape in human embryos and found that the denal lamina was anterioposteriorly flattened during the 6th week and began a progressive elongation and conformance to the catenary curve by 8 1/2 weeks. He also found more pronounced increases in intercanine distance as well as in the deptah of the arch anterior to the intercanine line. Brader[14] believed that the dental arch existed in harmony between the buccinator muscle, lip and tongue. He came to the conclusion that the ideal arch form should be tri-focal ellipse. In 1956, Hayashi[56] used a mathemetical formula to describe the dental arch. This formula was the first to give a versatile description of the dental arch.

Lu[57], in 1964, did a computor analysis of occlusion and claimed that the dental arch could be described by a 4th degree polynominal equation. Currier[59] analyzied the human dental arch and concluded that the ellipse had a better goodness of fit to the outer curve, upper and lower, than to the middle or inner curves. The parabola had a better goodness of fit to the middle curve in the upper and lower arches than to the outer or inner curve. Biggerstaff[62] thought a quadratic equation could best describe arch form(s), and which included the ellipse, parabla, and hyperbola. In 1975 a 6th degree polynominal equation was suggested[7] by Pepe. Recently cubic spline function was used by

BeGole[51] to described dental arch form and Sampson[52] used conic sections to analyze the normal dental arch and its variability.

Schulhop[72,65] designed a biparameter catenary curve from the lower arch to estimate individual (ideal?) arch form. The vairables he used were anterior tooth size, intermolar arch width, denture position, and individual facial angle. Other studies[73,68] suggested that dental arch forms were genetically determined. The dimensions of arch widths were all greater in brachycephalics than dolichocephalics. Many researchers [53,55,60,74,75] studied the effect of muscles on dental arch form.

Herren[63] applied a graphic method, arcogramme, to compare misshapen arches to an ideal arch. Musich and Acherman[64] used a catenometer to support their theory that a catenary curve provided the best arch form. White[67] studied 24 adult patients with superior dental occlusion and no history of orthodontic treatment. He found that only 8 % of the Bonwill-Hawley arches could be considered a good fit to these occlusions while 52 % of these arches exhibited a poor fits. Brader arches had 12.5 % good fit, while catenaries had 27 % good fit. The Rocky Mountain Data System had no poor fit designs, but had only two arches that could be called good fitting. Only 6.25 % of these arches were considered symmetrical.

Felton[61] studied the instability in changing the pre-existing arch form. In his study almost 70 % of his sample showed significant long-term (6 to 9 year) posttreatment changes. By using the finite element analysis, Baluta[15] was able to differentiate size changes from shape changes. He found that the shape changes were greatest in the non-extraction sample as compared with the extraction sample. Size changes resulting from orthodontic treatment were greatest in the first bicuspid extraction sample. (Table 1 shows a summary of available dental arch form studies.)

Table 1 - summarize dental arch studies

Author/year	variable	math/shape	data point	sample	ideal
Hawley/05	teeth	arc of a circle (3-3) line (6-3) toe in (7-6)	incisal and buccal cusp tip		yes
William/17	teeth	arc of circle center mid point buccal groove of first molars	outer		yes
Izard/27	none	ellipse, parabola, U			no
Lasher/34	muscle occlusion	circle	cusp tip incisor edge	none	yes
Hayashi/56	none	ax^{4}(hemisphere ovoid, ellipse, U-shape, parabol hyperbol	upper, incisal buccal cusp	56	no
Lu/64		4th degree polynominal			no
Burdi/68	none	catenary	enamel organ	lower 15 embryos 8 1/2 weeks	yes/no
Currier/69	muscle occlusion	ellipse(outer) parabola(middle)	outer, middle, inner upper and lower	25 good occlusion	yes
Biggerstaff/72	none	Quadratic equation (ellipse hyperbolic parabolic)	cusp tip incisor edge	small	no
Brader/72	muscle	tri-focal ellipse	outer upper outer lower	skull upper	yes
Hennen et al./73	none	open polygon	contact points		no
Pepe/75	none	6th degree polynominal	contact point	7 children	no
Schulof/75	tooth size facial angle arch width, denture position t	biparameter catenary			yes
White/78	none	follow natural arch perimeter			no
BeGole/80	none	cubic spline	upper incisor upper buccal cusp	27 Class I	no
Sampson/81	none	2nd degree polynominal	upper arch	66 Class I	no

Materials and Method

1. Sampling

The sample consisted of casts of 33 mandibular dentitions which were collected from the files of the Oregon Child Study Clinic and from records of the Orthodontic Department of the Oregon Health Science University. Three patients had a history of cleft palate. No case had received orthodontic treatment when the initial cast was taken and the mandibular arches were grouped according to their relationships to the upper arch.

Group A (fig. 5) had at least 1mm overbite and possessed anterior teeth contact in centric occlusion as well as positive cuspid overlap.

Group B (fig. 6) had at least 1mm overbite, anterior teeth contact in centric occlusion and spacing in the upper anterior segment (from distal proximal of cuspid to distal proximal of the cuspid on the opposite side). Likewise, positive cuspid overlap was present in Group B.

Group C (fig. 7) had both anterior and posterior crossbite. All cleft palate patients were included in this group.

Five subjects from the patient pool of the Orthodontic department of the Oregon Health Sciences University comprised

Group D (Fig. 8). This group had positive posterior overlap and zero to negative cuspid overlap.

Alginate impressions were taken of each lower cast. The impressions were photocopied and the contact points of each arch were marked. A line was drawn to connect all contact points and this line was analyzed in the following manner.

2. Test for stability

The lines were enlarged by a factor of two. Significant thickness of a lower incisor was artificially set at 6 mm, after enlargement. Arch stability was then determined by the graphic method suggested by G. T. Snelling[71] (appendix 3). However: the arch was considered stable if the line of resistance fell within the confinement of an arch instead of within the middle third.

Two individual conditions were tested: arch loading which led to mesial drift and arch loading which led to distal drift. Each arch was then classified stable or non-stable based on the above tests.

3. Statistical analysis

The Z score was used to test the hypothesis that the probabilities of the occurence of a stable result was the same for all groups

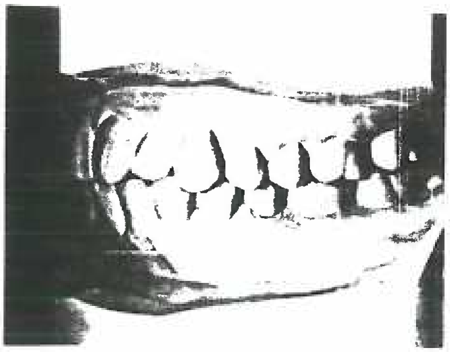


Figure 5. The occlusal pattern of Group A

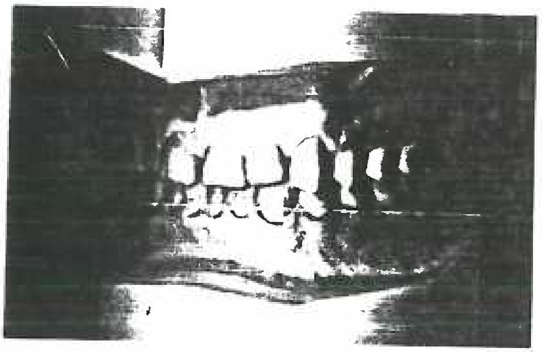


Figure 6. The occlusal pattern of Group B

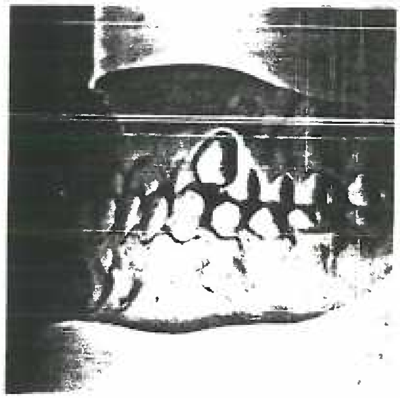


Figure 7. The occlusal pattern of Group C

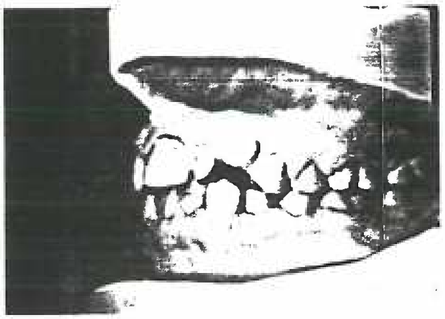
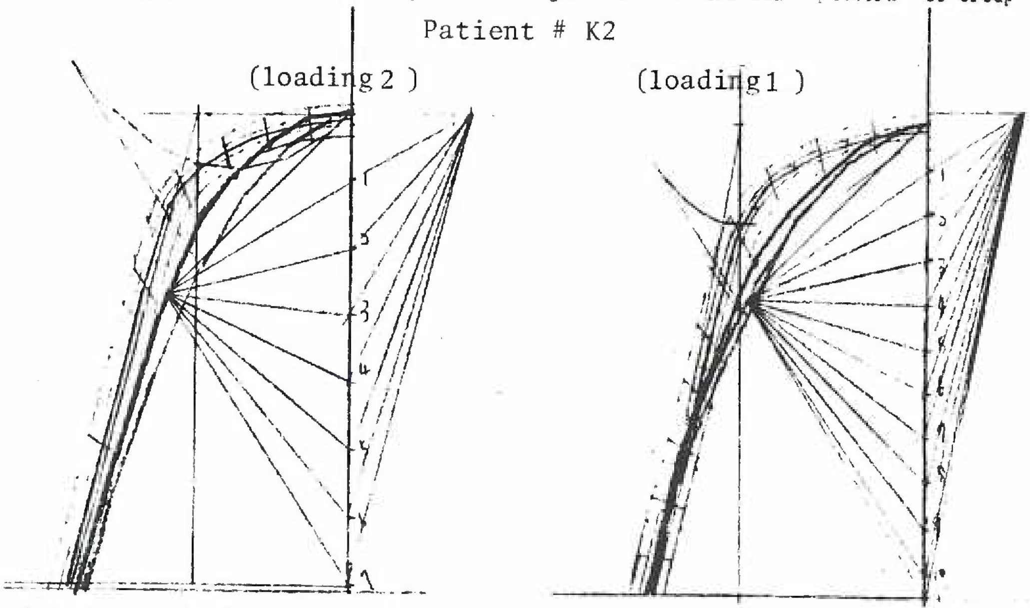


Figure 8. The occlusal pattern of Group D



Patient # K2

(loading 2)

(loading 1)

Figure 9, typical arch form and its line of resistance for Group D

Results

1. All the mandibular arches in Group A, B and C were stable under loading conditions which led to mesial drift. Five of the six arches in Group D were not stable under loading conditions for mesial drift.

2. For loading conditions which led to distal drift (Table 2), the probability of the curve of resistance being located within the confinement of the arches was higher in group A. This probability was statistically significant to the 95 % confidence interval.

Group D showed no stability in any condition. Overturn of the arch seemed to be an unavoidable situation. Typical arch form and its line of resistance for group D is shown in Fig 9.

Table 2- Statistical analysis of a loading condition which leads to distal drift

	within	outside	total
Group A	9	3	12
Group B	3	8	11
Group C	1	4	5
Group D	0	5	5
			33

H_0:p1=p2
p1=0.75, q1=0.25 p2=0.27, q2=0.73
σ =0.183
95 % (p1-p2)=0.48 ±0.35
reject null hypothesis

H_0:p1=p3
p1=0.75, q1=0.25 p3=0.20 ,q3=0.80
σ =0.22
95 % (p1-p3)=0.55 ±0.43
reject null hypothesis

H_0:p1=p4
p1=0.75, q1=0.25 p4=0.00, q4=1.00
σ =0.125
95 % (p1-p4)=0.75 ±0.12
reject null hypothesis

H_0:p2=p3
p2=0.27, q2=0.73 p3=0.20, q3=0.80
σ =0.22
95 % (p2-p3)=0.07 ±0.43
accept null hypothesis

H_0:p2=p4
p2=0.27, q2=0.73 p4=0.00, q4=1.00
σ =0.134
95 % (p2-p4)=0.27 ±0.13
reject null hypothesis

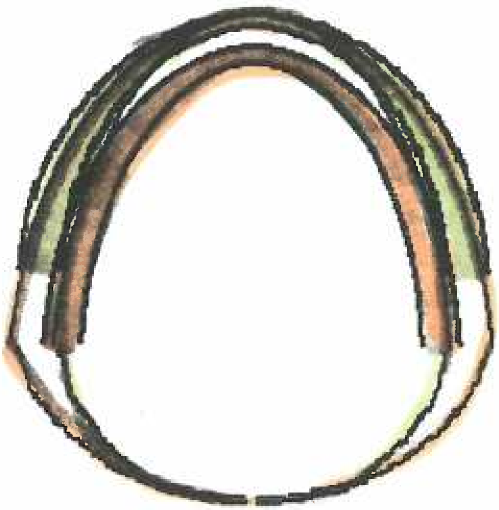
H_0:p3=p4
p3=0.20, q3=0.80 p4=0.00, q4=1.00
σ =0.18
95 % (p3-p4)=0.20 ±0.18
reject null hypothesis

Discussion

From a review of the literature an anterior component of occlusal force seems to exist. This force creates mesial migration of the dentures. In contrast, the presence of a distally directed force toward the lower denture during centric biting seemes to be obscure. According to O'Leary[69], the majority of the upper anterior teeth did not have satisfactory proximal contacts on both the mesial and distal aspects. Our clinical experience tells us that if a diastema is present it usually occurs in the upper anterior segment. Likewise, the lower anterior segment is the most frequent site of dental arch crowding. These observations suggest that the lower anterior segment might have sustained a distally directed force (compression) and the upper anterior segment a mesial directed force (tension).

Futher evidence to support a distal force theory comes from the fact that the posterior teeth possess heavy contact in centric occlusion while anterior teeth possess light contact in centric occlusion. Light contact of the anterior teeth in centric occlusion allows some freedom for the jaw to move forward. This forward movement of the lower jaw might maintain the light contact on the anterior teeth since as the lower jaw moves forward the anterior teeth have to be driven back.

of premolars and molars and how they fit closely into the concave surface of the adjacent tooth. (copy from Beyron)

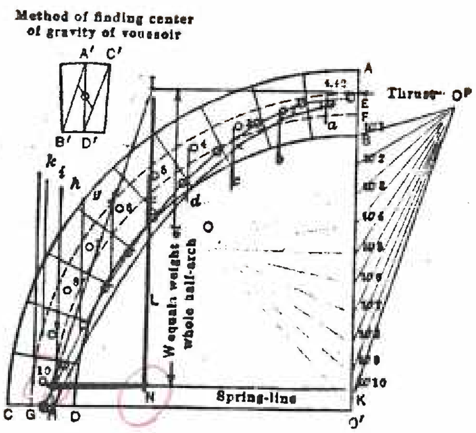


thin line: buccinator sling

thick line: dentures

yellow: before loading

green: after loading



Lateral thrust from the system ON

Figure 11. Differential movement of teeth during function (centric biting)

This study assumed that the mesial driving force and distal driving force were the two main forces in maitaining the stability of the dental arch. The arches had to be stable as long as these forces existed. Under this assumption the four groups were chosen. As we know the mesial force is not entirely related to the occlusion, it is also related to the axial inclination of the teeth and existence of occlusal contact. However, the distal force is related to the contacting relationship of the upper and lower arches. The tighter the upper arch, the more resistance it affords to the lower arch, when the lower arch moves forward. The groups were differentiated according to the relationship of upper to lower anterior teeth.(See figure 12)

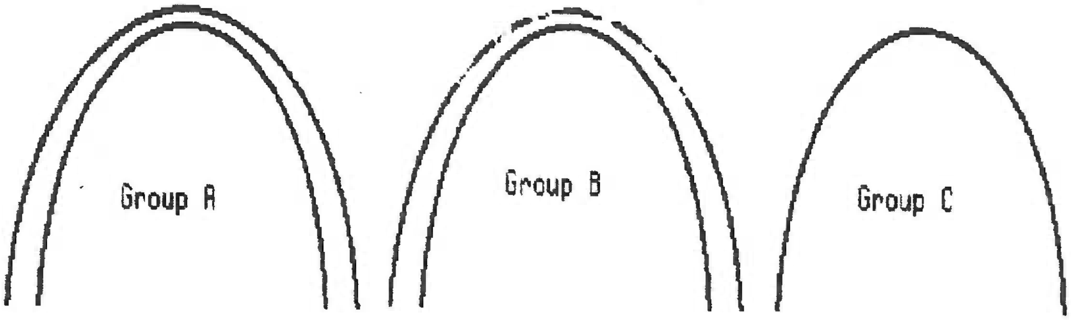


Figure 12, the mechanical models of groups A, B and C, differentiated from the limitation of upper dentition toward the lower's

If the proposed theory was valid, then all the arches in groups A, B and C should be stable under mesial forces and those in group A should be stable under distal forces. The findings show that few arches in groups B and C were stable under distal forces. Group D was chosen because these arches possess a breaking point (no cuspid contact) in the arch. The typical arch form of group D was U shaped and it could never be considered stable under either type of loading. This finding implies that the cuspids play an important role in the maintainance of arch stability. Of course, further research is required to confirm this statement.

In analyzing this research one can see that six millimeters of effective thickness, after enlargement, was chosen for a lower incisor. For a force producing mesial drift in both arches this thickness should be larger in order to maintain arch stability. This distance was supposed to be the distance between the most lateral anterior teeth contact (either occlusal or proximal) and the most medial anterior teeth contact. For a force producing distal drift in the mandibular arch, three millimeters of actual thickness of the lower arch seemed excessive. But one must remember that the effective thickness was arbitrarily chosen which make this distance remain the same whether the teeth were rotate or not.

Likewise, the confinement chosen to decide stability of an arch is questionable. In engineering, if the curve of resistance was within the middle third of an arch, the arch was considered stable. In this study the position of the line of resistance had to fall within the arch which again was an arbitrarily decision. However, when a cable and an arch lay against each other, the effective thickness of an arch may be larger than it was chosen by the engineer.

A computer was used to fit caternary and parabolic curves onto the arch and/or the line of resistance of the arch. No particular pattern could be ascertained. This finding might suggest that the 'ideal' arch form did not exist and that the only thing that matters is 'arch stability'. For the same reason, the non-parametric statistical analysis which was used in this study could be all that was necessary.

Summary and conclusion

A theory of arch stability was presented in this study. The mesially and distally directed forces were used to determine the arch stability. There were four groups of subjects in this study, grouped according to the relationship between the upper and lower arches.

Lower arches which had a greater influence from the upper arch (i.e. anterior tooth overlap and strength of the upper arch) exhibited a greater number of characteristies of distal drift. Although forces of occlusion, occlusion, skeletal relationships, and the musculature were thought to be important in creating a balanced position of the arch, only the relationship of occlusion to arch atbility was studied. No ideal arch form was known for each individual and further study was needed.

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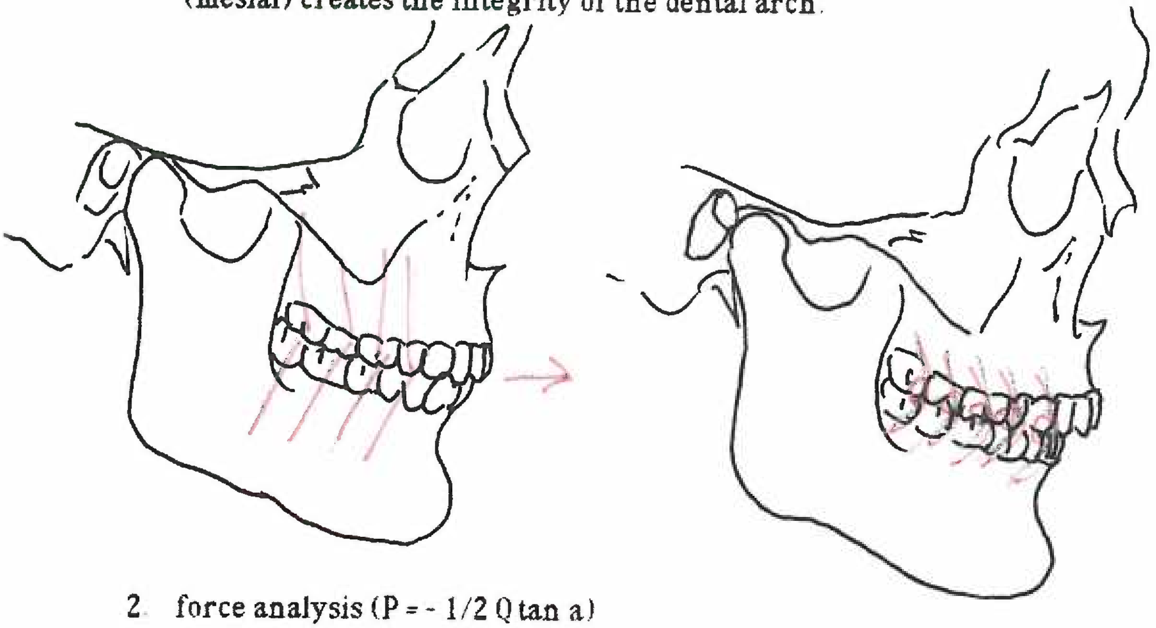
Appendix 1

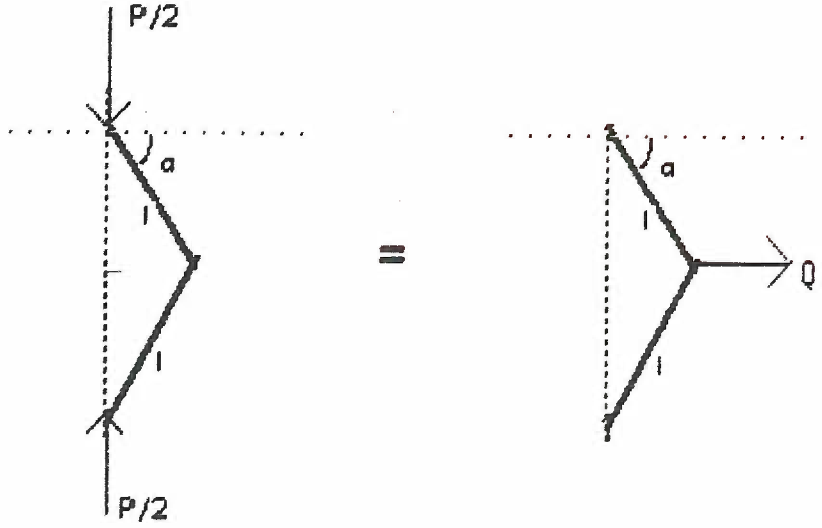
Mesial drift

1. a spring

The mesial inclination of the teeth in the posterior segments creates a spring effect in the masticatory system. Since the mandible is not a rigid body, it will deform whenever a force is applied to it. If the mandible is fixed tightly at one end (occlusal contact), the stress has to express itself in another dimension (the joint). And the joint is not designed to sustain torsion. (Figure). This force (mesial) creates the integrity of the dental arch.



2. force analysis (P = - 1/2 Q tan a)

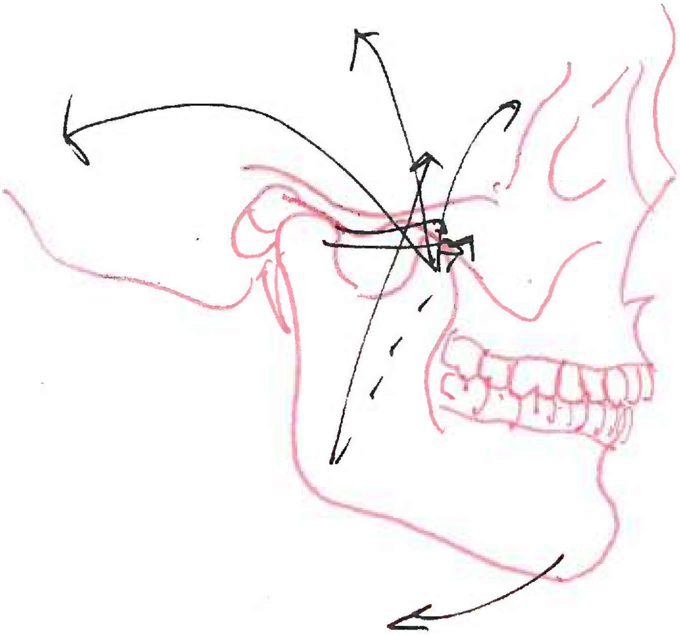


Appendix 2

Disal drift

1. The masticatory system

The masticatory system is a machine. It is composed of two joints (right and left temoromandibular joints), two compartments (craniomaxillary complex, and mandible) and muscles. The two main function of the masticatory muscles are, first, to produce syncronized motion of the joints and, second, to create force to process food. Syncronization is crucial.



The upper figure is the sagital view of the masticatory system. Red strings will be used to indicate various directions of muscle pull which are thought to be important in doing a functional analysis.

2. System analysis (to analyze the forces which are created by isometric muscle contraction when the teeth are in the maximum intercuspation.) The similarity between masticatory system and a loaded spring was described in figure 2 to figure 3.

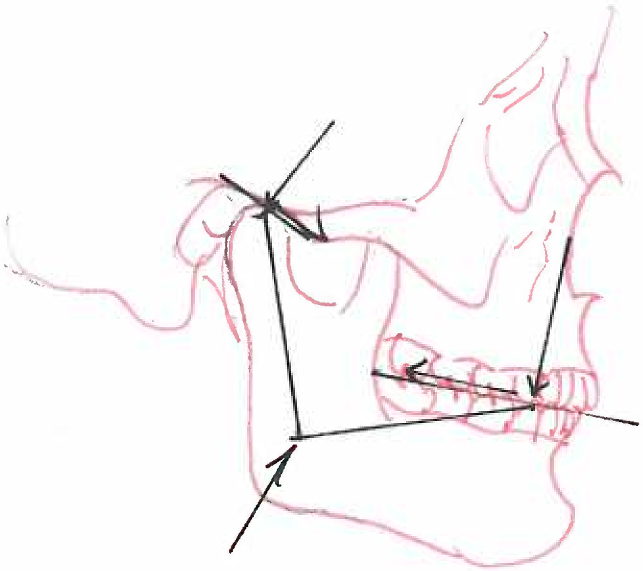


Figure 2

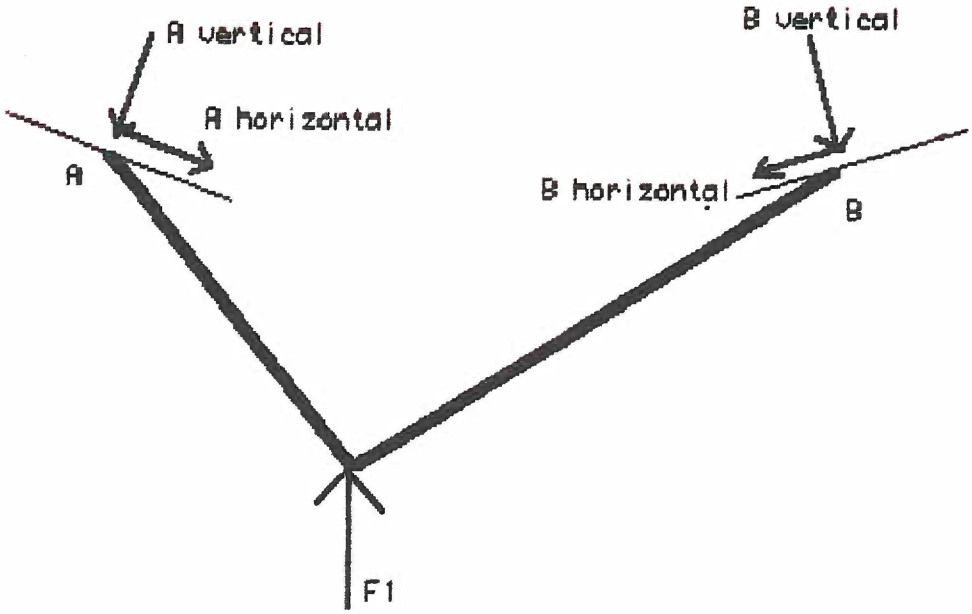


Figure 3

One of the inclines in figure 3 (for example incline plane A) simulates condylar guidance or articular eminence in figure 2. The other incline (B) corresponds to the occlusal plane. The modulus of elasticity of a spring will have the same meaning as that in the mandible. This characteristic will add to some of the horizontal factors in resisting the spreading action of the mandible during loading. Likewise, the action of the lateral pterygoid muscle will prevent some sliding movement of the mandbile along the incline (articular eminence), and the horizontal and vertical overlap of the upper denture toward the lower will inhibit it from sliding to the other direction.

See figure 4. According to the graph, the following conclusions can be made. First, the further the center of occlusal forces away from the location of muscle insertion, the stronger the pressure that will be created in the temporomandibular joint. Second, the steeper the articular eminence the tighter the lateral ptergoid has to pull in order to create stability. Third, the flatter the occlusal plane, the more forward thrust which can be prevailed. Fourth, the tighter the occlusal interlock, (i.e. deep overbite) the more freedom which is needed in the joint (when B point in figure 5 represents the occlusion). Fifth, the tighter the joint (lateral pterygoid) the more freedom needed in centric occlusion. In this case B point in figure 5 represents the joints. Sixth, the smaller the curvature of the mandible, the more horizontal component of the occlusal force which can be prevailed. This characteristic is simulated by the spring constant in figure 5.

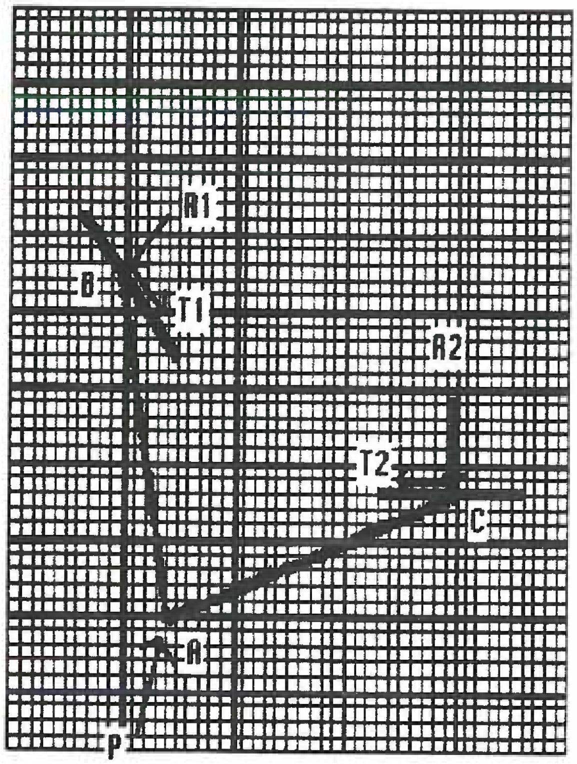


Figure 4

A: insertion of masseter

B: condyle and articular eminence

C: center of the occlusal force and occlusal plane

P: masticatory force

R1: resistance from eminence

R2: resistance from upper dentition

T1: pull from the lat. pterg. and mandibular stress

T2: occlusal resistance from upper dentition and mandibular stress

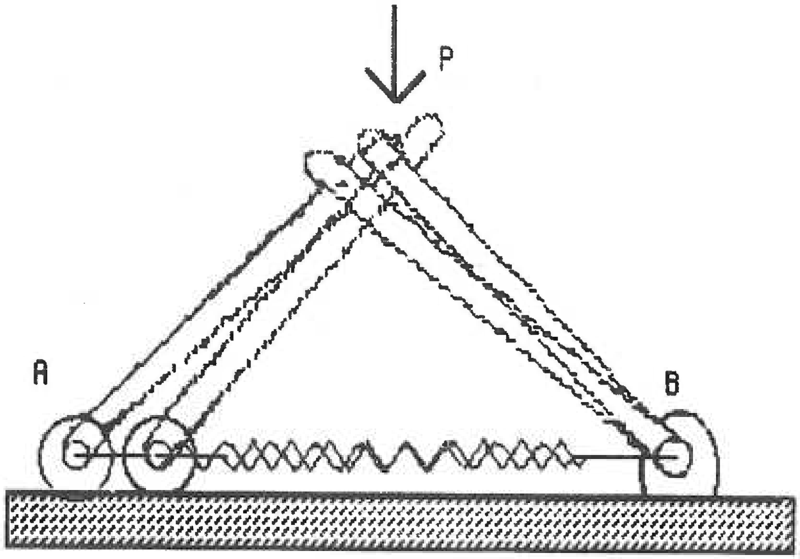


Figure 5

First Step. Draw one-half the arch to as large a scale as convenient, and divide it into voussoirs of equal size. In the example shown in Fig. 12, the arch-ring is divided into ten voussoirs of equal face-areas. As already pointed out, it is not necessary that these should represent the actual voussoirs of which the arch is built. Next, the face-area of each of these voussoirs is to be found. Where the arch-ring is divided into voussoirs of equal size, this is most easily done by computing the total area of the arch-ring and dividing this total area by the number of voussoirs. The FORMULA for finding the area of one-half the arch-ring is as follows:

$$\text{Area in square feet} = 0.7854\ (r^2 - r_1^2)$$

In this formula r is the outside radius and r_1 the inside radius in feet.

In this problem, for example, if the

$$\text{Area of the arch-ring} = 0.7854\ (12.5^2 - 10^2) = 44.2 \text{ sq ft}$$

as there are ten equal voussoirs, the area of each voussoir is 4.42 sq ft. Having drawn out one-half of the arch-ring, divide the crown-joint into three equal parts, and with radii of $O'E$ and $O'F$ describe the arcs dividing the arch-ring into thirds.

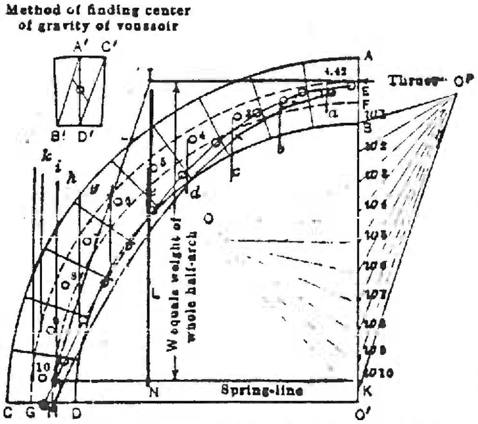


Fig. 12. Line of Pressure in Unloaded Semicircular Arch-ring

Second Step. Choose the points E and H through which to pass a MINIMUM LINE OF RESISTANCE. The points F and G, through which a MAXIMUM LINE OF RESISTANCE can be passed, could equally well have been chosen. It should be noted that an unloaded semicircular arch is more apt to fail by opening at the intrados at the crown and at the extrados at the haunch, and therefore, in this case, the line of resistance probably passes nearer the OUTER THIRD at the CROWN and nearer the INNER THIRD at the HAUNCH. To determine this MINIMUM LINE OF RESISTANCE the MINIMUM THRUST, applied at the point E of the crown-joint, must first be determined.

The half-arch is in equilibrium under the action of three forces: (1) the THRUST AT THE CROWN, acting horizontally, applied at the point E and preventing the half-arch from overturning inward; (2) the WEIGHT OF THE HALF-ARCH considered as a vertical force, acting through its center of gravity and tending to overturn it inwards about the point D; and (3) A FORCE EQUAL AND OPPOSITE TO THE RESULTANT of these two forces and passing from H to I. I is the intersection of the weight-line through the center of gravity of the half-arch, with the line of action of the thrust at the crown, prolonged. It is thus possible to construct the TRIANGLE OF THESE THREE FORCES and determine the magnitudes of the thrusts, when the position of the weight-line of the half-arch is determined. It is first necessary to draw a vertical line through the center of gravity of each voussoir. The center of gravity of one of the voussoirs may be found by the METHOD OF TRIANGLES, as shown in the supplementary figure at the side of the arch-ring.

Having determined the positions of the centers of gravity of the voussoirs, locate them on the voussoirs as shown. From the point E (Fig. 12) lay off vertically, to a scale of so many SQUARE UNITS TO A LINEAR UNIT, the area of each voussoir, one below the other, commencing with the top voussoir. The length of the line EK will then equal the total area of the arch-ring. From E and K (Fig. 12) draw 45° lines intersecting at O. Draw Ow 1, Ow 2, Ow 3, etc. Then where OE intersects the first vertical line through the center of gravity of the first voussoir at a, draw a line parallel to Ow 1, intersecting the second vertical at b. Draw bc parallel to Ow 2, cd parallel to Ow 3 and so on to k. Draw kL parallel to Ow 10 and prolong it downward until it intersects EO prolonged, at L. A vertical line drawn through L will pass through the center of gravity of the half arch-ring. This is an application to a practical problem of the method of finding, by the EQUILIBRIUM-POLYGON, the line of action of the resultant of a SYSTEM OF PARALLEL FORCES. The weights of the individual voussoirs act along parallel vertical lines and the weight of the half-arch is their resultant in magnitude.

Third Step. To determine the THRUST AT THE CROWN and the REACTION AT THE SPRING, draw a horizontal line through E, the upper part of the middle third, and a vertical line through L, the two lines intersecting at I (Fig. 12). For the arch to be stable, it is, in general, considered necessary for the LINE OF RESISTANCE to pass within the MIDDLE THIRD. First, assume that the line of pressure or resistance starts at E and comes out at H. Draw a line IH the direction of the line of action of the resultant of the thrust at the crown and the weight of the half-arch, and draw, also, a horizontal line opposite the point w 10, between N and M. This horizontal line MN represents the magnitude of the horizontal thrust at the crown, for INM is the TRIANGLE OF THE THREE FORCES in equilibrium, the THRUST at the crown, the WEIGHT of the half-arch and the REACTION at the spring. Draw w 10 O^P parallel to HI, and the lines O^Pw 1, O^Pw 2, O^Pw 3, etc. O^PE, equal to NM, is the thrust at the crown, and w 10 O^P, equal to MI, the reaction at the spring. INM and EKO^P are similar triangles.

Fourth Step. It is required next, to determine the LINE OF RESISTANCE through the arch-ring. The thrust at E is combined with the weight of the first voussoir; their resultant is found and in turn combined with the weight of the second voussoir; and so on for all the voussoirs. The intersections of these resultants with the joint-lines are the CENTERS OF PRESSURE; the line joining these centers of pressure is the LINE OF RESISTANCE.

These resultants could be determined by drawing a series of PARALLELOGRAMS OF FORCES over each voussoir. This would complicate the figure and involve unnecessary labor. It is found more convenient to draw the TRIANGLES OF FORCES one after the other, at the right-hand side of the figure and then transfer the results thus obtained by means of parallel lines to the figure itself, especially as the weights of the voussoirs have already been laid off along the line EK, at Ew 1, w 2, w 3, w 4, w 5, etc.

Then from the point where O^PE prolonged intersects the first vertical in voussoir number 1, draw a (green) line to the second vertical, parallel to O^Pw_1; from this point, a (green) line to the third vertical, parallel to O^Pw 2 and so on. The last line should pass through H. Join the various points, where these (green) lines cut the joints at the centers of pressure, by the broken (red) line. This last line drawn is the LINE OF RESISTANCE. If this line lies entirely within the MIDDLE THIRD of the arch-ring, the arch may be considered to be stable. But suppose that the line of resistance passes not only outside of the middle third but also outside of the arch-ring itself; it is still possible that the arch is not unstable. This is the case in Fig. 12 and we will

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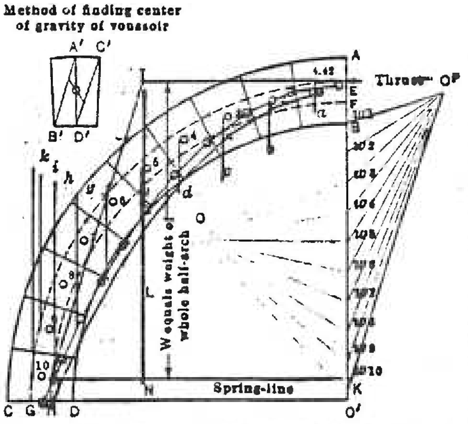

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Second Step. Choose the points E and H through which to pass a MINIMUM LINE OF RESISTANCE. The points F and G, through which a MAXIMUM LINE OF RESISTANCE can be passed, could equally well have been chosen. It should be noted that an unloaded semicircular arch is more apt to fail by opening at the intrados at the crown and at the extrados at the haunch, and therefore, in this case, the line of resistance probably passes nearer the OUTER THIRD at the CROWN and nearer the INNER THIRD at the HAUNCH. To determine this MINIMUM LINE OF RESISTANCE the MINIMUM THRUST, applied at the point E of the crown-joint, must first be determined.

The half-arch is in equilibrium under the action of three forces: (1) the THRUST AT THE CROWN, acting horizontally, applied at the point E and preventing the half-arch from overturning inward; (2) the WEIGHT OF THE HALF-ARCH considered as a vertical force, acting through its center of gravity and tending to overturn it inwards about the point D; and (3) A FORCE EQUAL AND OPPOSITE TO THE RESULTANT of these two forces and passing from H to I. I is the intersection of the weight-line through the center of gravity of the half-arch, with the line of action of the thrust at the crown, prolonged. It is thus possible to construct the TRIANGLE OF THESE THREE FORCES and determine the magnitudes of the thrusts, when the position of the weight-line of the half-arch is determined. It is first necessary to draw a vertical line through the center of gravity of each voussoir. The center of gravity of one of the voussoirs may be found by the METHOD OF TRIANGLES, as shown in the supplementary figure at the side of the arch-ring.

Having determined the positions of the centers of gravity of the voussoirs, locate them on the voussoirs as shown. From the point E (Fig. 12) lay off vertically, to a scale of so many SQUARE UNITS TO A LINEAR UNIT, the area of each voussoir, one below the other, commencing with the top voussoir. The length of the line EK will then equal the total area of the arch-ring. From E and K (Fig. 12) draw 45° lines intersecting at O. Draw Ow 1, Ow 2, Ow 3, etc. Then where OE intersects the first vertical line through the center of gravity of the first voussoir at a, draw a line parallel to Ow 1, intersecting the second vertical at b. Draw bc parallel to Ow 2, cd parallel to Ow 3 and so on to k. Draw kL parallel to Ow 10 and prolong it downward until it intersects EO prolonged, at L. A vertical line drawn through L will pass through the center of gravity of the half arch-ring. This is an application to a practical problem of the method of finding, by the EQUILIBRIUM-POLYGON, the line of action of the resultant of a SYSTEM OF PARALLEL FORCES. The weights of the individual voussoirs act along parallel vertical lines and the weight of the half-arch is their resultant in magnitude.

Third Step. To determine the THRUST AT THE CROWN and the REACTION AT THE SPRING, draw a horizontal line through E, the upper part of the middle third, and a vertical line through L, the two lines intersecting at I (Fig. 12). For the arch to be stable, it is, in general, considered necessary for the LINE OF RESISTANCE to pass within the MIDDLE THIRD. First, assume that the line of pressure or resistance starts at E and comes out at H. Draw a line IH the direction of the line of action of the resultant of the thrust at the crown and the weight of the half-arch, and draw, also, a horizontal line opposite the point w 10, between N and M. This horizontal line MN represents the magnitude of the horizontal thrust at the crown, for INM is the TRIANGLE OF THE THREE FORCES in equilibrium, the THRUST at the crown, the WEIGHT of the half-arch and the REACTION at the spring. Draw w 10 O^P parallel to HI, and the lines O^Pw 1, O^Pw 2, O^Pw 3, etc. O^PE, equal to NM, is the thrust at the crown, and w 10 O^P, equal to MI, the reaction at the spring. INM and EKO^P are similar triangles.

Fourth Step. It is required next, to determine the LINE OF RESISTANCE through the arch-ring. The thrust at E is combined with the weight of the first voussoir; their resultant is found and in turn combined with the weight of the second voussoir; and so on for all the voussoirs. The intersections of these resultants with the joint-lines are the CENTERS OF PRESSURE; the line joining these centers of pressure is the LINE OF RESISTANCE.

These resultants could be determined by drawing a series of PARALLELOGRAMS OF FORCES over each voussoir. This would complicate the figure and involve unnecessary labor. It is found more convenient to draw the TRIANGLES OF FORCES one after the other, at the right-hand side of the figure and then transfer the results thus obtained by means of parallel lines to the figure itself, especially as the weights of the voussoirs have already been laid off along the line EK, at Ew 1, w 2, w 3, w 4, w 5, etc.

Then from the point where O^PE prolonged intersects the first vertical in voussoir number 1, draw a (green) line to the second vertical, parallel to O^Pw_1; from this point, a (green) line to the third vertical, parallel to O^Pw 2 and so on. The last line should pass through H. Join the various points, where these (green) lines cut the joints at the centers of pressure, by the broken (red) line. This last line drawn is the LINE OF RESISTANCE. If this line lies entirely within the MIDDLE THIRD of the arch-ring, the arch may be considered to be stable. But suppose that the line of resistance passes not only outside of the middle third but also outside of the arch-ring itself; it is still possible that the arch is not unstable. This is the case in Fig. 12 and we will next determine if a line of resistance can be drawn which will remain within the limits of the middle third of the arch-ring.

Fifth Step. The Second Trial. Reproducing the condition of Fig. 12 in Fig. 13, without the construction lines, it is seen that the LINE OF RESISTANCE leaves the arch-ring at R and enters it again at S, while it is furthest from it at U. If, at U, a perpendicular is erected to a straight line joining the two points R and S, this perpendicular line VW, called the LINE OF FRACTURE, will be approximately the trace of the plane along which, with the line of resistance under consideration, the arch will tend to fail, presumedly by TURNING OVER to the right about the point V. This shows that the THRUST AT THE CROWN, assumed to be applied at the point E, while of sufficient intensity to maintain equilibrium about H, is not of sufficient intensity to maintain equilibrium about V. If now a SECOND THRUST, of sufficient intensity to maintain equilibrium about V, or better, about X, can be applied at E without being so great in magnitude that it will OVERTURN THE ARCH OUTWARD about G, or some other point on the outer line of the middle third, it is reasonable to conclude that the line of resistance resulting from this thrust is very nearly the TRUE LINE OF RESISTANCE in the arch-ring and that the arch is stable.

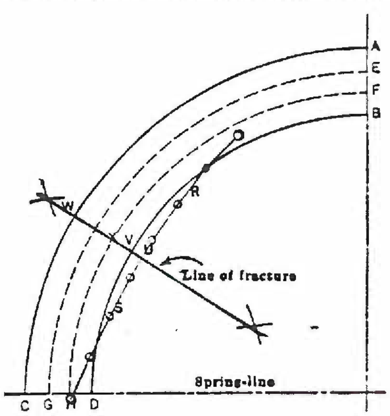

Fig. 13. Line of Fracture in Unloaded Semicircular Arch-ring

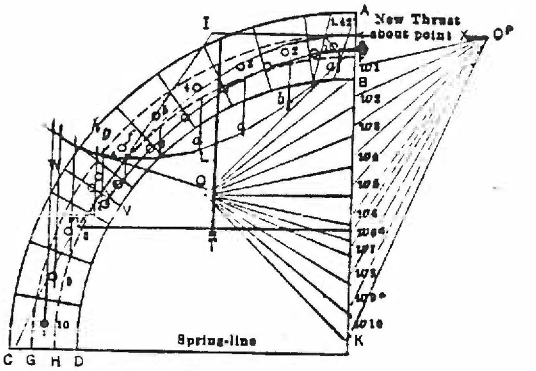

Fig. 14. Second Line of Pressure in Unloaded Semicircular Arch-ring

In order to determine this NEW LINE OF RESISTANCE the NEW THRUST AT THE CROWN must be found (Fig. 14). The preliminary steps required for this are the same as before until the seventh voussoir is reached. This is divided into two voussoirs by the line VW (Fig. 14), one being $w6\ w6^a$ and the other the remainder of this seventh voussoir, and this division must be allowed for along the load-line EK, at $w6\ w6^a$. The line $w6\ w6^a$ represents the area of voussoir 6^a, and the line $w6^a\ w7$ the area of the remainder of the seventh voussoir.

The vertical line IL, passing through the center of gravity of that part of the half-arch above the line VW, is found by prolonging backwards the line hg, parallel to $O\ w6^a$, until it intersects OE at L. To find the NEW THRUST AT THE CROWN by completing the TRIANGLE OF FORCES for this thrust and the force equal and opposite to their resultant, the inclined (blue) line must be drawn through the point X and the horizontal (blue) line through $w6^a$. The new thrust then is as before NM, equal to O^PE. This thrust is laid off at O^PE, the (green) lines O^Pw 1, O^Pw 2, O^Pw 3, etc., being drawn as before and the new line of resistance being drawn through the points where the parallels to these

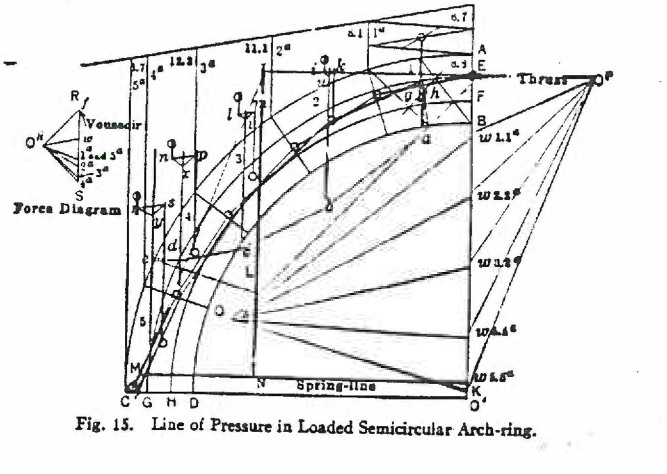

Fig. 15. Line of Pressure in Loaded Semicircular Arch-ring.

(green) lines cut the joints. This NEW LINE OF RESISTANCE, if drawn correctly, should pass through X. It lies within the middle third, except for a short distance at the springing, and hence it is justifiable to consider the arch stable. If it had passed outside the middle third to any great extent, in this second trial, this presumption would not have been justified.

This discussion explains the method of determining the stability of an UNLOADED SEMICIRCULAR ARCH. Such cases very seldom occur in practice, but they serve to illustrate the methods which apply generally to all other cases. With LOADED ARCH-RINGS there is slight difference in the method of determining the position of the center of gravity.

Example 3. A LOADED OR SURCHARGED SEMICIRCULAR ARCH (Fig.15) will be considered next. Assume the same arch shown in Figs. 12, 13 and 14, and suppose it to be loaded with a wall of masonry of the same thickness and weight per square foot as that of the arch-ring, the upper surface of the wall being an inclined plane, 1 ft above the arch-ring at the crown, and 8 ft above it at the spring. The assumption of the particular load in this case is a purely arbitrary one for the purpose of illustrating the method of solution. The determination of the ACTUAL LOAD that comes upon an arch in any given case is by no means easy, so numerous are the uncertain elements that affect the transmission of this load to the arch-ring.

The customary procedure is to assume that the load is itself transmitted to the arch-ring VERTICALLY DOWNWARD. Each voussoir thus receives that portion of the load which is included between two vertical lines drawn to the points of intersection of the joints on either side of that voussoir with the extrados. Having made this assumption it is necessary next to determine how much of the total superimposed masonry bears upon the arch-ring.

It is a matter of common observation that if an opening is made in a wall, especially in a wall that has stood for some time, the major portion of the masonry above this opening is self-supporting, limited portions only, bounded by a somewhat irregular line, falling down into the opening, as shown in Fig. 16. The profile of this boundary-line depends upon the nature of the material of which the wall is constructed, the size of the stones, bricks, etc. the character of the bond and the quality of the mortar. This being the case, all the masonry above an arch should not be considered as the load on it. Some authorities recommend considering as the proper load, for brickwork, a TRIANGULAR PART of the wall, the sides of which triangle have an inclination to the horizontal of 45°; others assume an inclination of 60° (Fig. 16). The exact determination of this load by mechanical laws is difficult if not impossible. It is better to consider each case separately and by a careful study of the conditions to determine as closely as possible just what portion of the weight of the superimposed masonry is transmitted to the arch. Having assumed a load for this particular arch-ring (Fig. 15), the procedure is as follows:

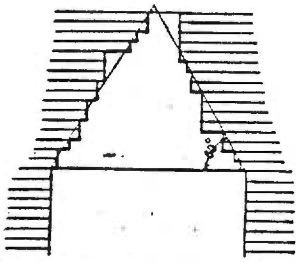

Fig. 16. Triangle of Loading over Opening

First Step of Example 3. This involves the finding of the CENTER OF GRAVITY of the ARCH-RING AND LOAD COMBINED. Divide the arch-ring into five oussoirs of equal size. In this case the area of each voussoir is equal to 4.2 sq ft ÷ 5, or 8.8 sq ft. (See under First Step, Fig. 12, preceding example.) The surcharge or load, also, is divided into five parts, not necessarily qual, by drawing vertical lines to the points of intersection of the joints and he extrados. The approximate area of each one of these surcharges is found y multiplying half the sum of the lengths of the two parallel vertical sides y the length of the horizontal distance between them.

The positions of the center of gravity of each voussoir and of the center of ravity of each voussoir-surcharge are determined as in the preceding example. The CENTERS OF GRAVITY of these SURCHARGES can be found by dividing ch TRAPEZOIDAL FIGURE into TRIANGLES as shown, remembering that the EDIAL LINE in this case joins the middle points of the two parallel faces. s the latter are vertical, the medial lines approach a horizontal direction. his construction is shown on surcharge 1^a, Fig. 15. Having drawn the of action of the weights of the various voussoirs and of their loads th their respective centers of gravity, the lines of action of the combined t of each voussoir and its load must be found. The construction for this operation is shown at the left of Fig. 15. The method used, that of the EQUILIBRIUM-POLYGON, is the same as that employed in the previous example to find the line passing through the center of gravity of the half-arch, only in this case the forces are reduced to two. Furthermore, as the areas of the various voussoirs are equal it is possible to superimpose the different FORCE-DIAGRAMS, one over the other, and so save considerable labor. Begin, therefore, by laying off along the line RS at the left of the loaded arch, and at any convenient scale, fw, the area (weight) of a voussoir; then from w, in turn, the distances $w\ 1^a$, $w\ 2^a$, $w\ 3^a$, etc., representing the areas of the successive surcharges, 1^a, 2^a, 3^a, etc., always at the same scale. The scale to be employed later for laying off the combined weights of the voussoirs and their loads along the line AK is the best one to choose, but the difference in scales is not important. In this particular instance the two points 1^a and 5^a coincide because the two areas 1^a and 5^a, although of different shapes, are each equal to 6.7 sq ft. This is a mere coincidence. Next draw fO'' and $4^a\ O''$ at 45° to RS, and in turn, $O''w$, $O''1^a$, $O''2^a$, etc. As the problem which presents itself is to combine the weight of each voussoir with its individual surcharge, and as the weights of all the voussoirs are equal, and, furthermore, as the forces which are to be combined to find their resultant are only two, the two POLE-LINES OR RAYS $O''f$ and $O''w$ in the FORCE-DIAGRAM serve in each case, and the FUNICULAR POLYGON is reduced to a TRIANGLE. Draw gh, ik, lm, np and rs parallel to $O''w$, and ht, ku, mv, px and sy parallel to $O''f$; and draw gt, iu, lv, nx and ry parallel respectively to $O''1^a$, $O''2^a$, $O''3^a$, $O''4^a$ and $O''5^a$. The points t, u, v, x and y are the points through which to draw the heavy (red) lines of action of the combined weights of the voussoirs and their surcharges.

Having found and drawn these lines, the procedure for finding the line IN is the same as in the previous example, except that the distances $Ew1\ 1^a$, $w^1\ 1^a$, $w2\ 2^a$, etc., instead of being equal to the weights of the voussoirs alone, are equal to the combined weights of each voussoir and its surcharge, $Ew1\ 1^a$, being equal to $f\ 1^a$, $w1\ 1^a$ to $w2\ 2^a$ being equal to $f\ 2^a$, etc.

The line EO is drawn at 45° to AO', but as the position of the POLE-POINT, O, is entirely arbitrary, the line $Ow\ 5\ 5^a$ has been drawn in this case in such a way that O falls well over toward the left of the figure, thus avoiding a certain amount of confusion in the drawing which would have resulted if $Ow\ 5\ 5^a$ had made an angle of 45° with AO'. The lines ab, bc, cd and de are drawn respectively parallel to $w1\ 1^aO$, $w2\ 2^aO$, etc., and eL is produced backward parallel to $Ow\ 5\ 5^a$ until it intersects EO at L, which is the point through which the heavy (red) line IN, passing through the center of gravity of the whole half-arch and its surcharge, should be drawn. A vertical line drawn through L will pass through the center of gravity of the arch-ring and its load. If this were an arch designed for a building and if the only abutments possible were of such size and form that it was essential for the thrust exerted by the last or fifth voussoir on these abutments to approach more nearly the vertical, the architectural expedient of increasing slightly the weight of the surcharge, 5^a, on this voussoir by adding some piece of ornament, such as a cartouche, could be resorted to. A case of this kind in actual practice is the archway over the entrance to the service-courtyard of the Grand Opera House in Paris, where the pyramidal stone ornaments which surmount the cornice on either side of the central motive were added after the original design was made, with this end in view. In the example illustrated in Fig. 15 the areas of the faces of the surcharges are shown by the figures on these faces. For the second surcharge from the crown, for example, the area is 3.1 sq ft.

Second Step of Example 3. This involves the determination of the THRUST AT THE CROWN and the LINE OF RESISTANCE. The method of finding this thrust at the crown is similar to that employed in the previous example. In that example, however, it was found that this thrust, applied at E and determined by assuming H as the point of application of the reaction at the spring, produced a line of resistance which fell considerably below the middle third. But instead of performing the operations required by a second trial, as in the previous example, the expedient is tried of slightly increasing the inclination to the vertical of the (blue) line IM, and so assuming a somewhat greater THRUST AT THE CROWN. As the line of resistance, as shown in Fig. 15, passed with this thrust departs but slightly from the middle third near the springing, we are justified in assuming that this arch is stable under the given conditions. The method used for this example may be used, also, for a SEMIELLIPTICAL ARCH.

Appendix 4

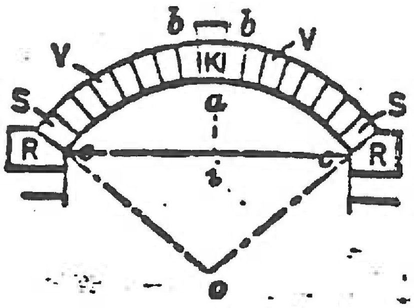

Fig. 1. Diagram of Segmental Arch

span: distance ec

rise: ai

crown: b

soffit or intrados: the lower boundary line eac

back or extrados: the outer boundary line

faces: the sides of the arch which are seen

voussoirs: the blocks of which the arch itself is composed are called voussoirs

Keystone: the center voussoirs

springers: the lowest voussoirs

skewback: In segmental arches, or those of which the intrados is not a complete semicircle, the springers generally rest upon two stones, as RR, which have their upper surfaces cut to receive springers; these stones are called skewbacks.

haundhes: the sides of the arch

springing-line: the line connecting the lower edges of the springers

spandrels: the loads in the triangular spaces, between the haunches and a horizontal line drawn from the crown

piers: The blocks of masonry, or other material, which support two successive arches

abutments: the extreme blocks which, in the case of stone bridges, generally support, on one side, embankments of earth, are called abutments centers of pressures of the joints: the points in which various thrusts cut the joints

line of pressure or line of resistance: the points in which these various thrusts cut the joints

overturn: if the center of resistance line lies outside the middle third, there will be a tendency for the voussoir to overturn

slide: If the angles made by the different thrusts with the normals to the joints are more than the angle of friction of the material of which the arch is constructed.