MATURATION CHANGES IN ORTHODONTICALLY UNTREATED HUMAN DENTAL ARCHES USING CUBIC SPLINES

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INTRODUCTION

Arch form has been a subject of more than academic interest since the philosophies of Tweed[1] and Strang[2] came into acceptance. Angle[3] had described the dental arch as being parabolic in form, and he acknowledged that the form for each individual varied according to race, type, temperament, etc. However, the philosophy of Angle was heavily weighted with the requirement that no dental units be removed:

> "The best balance, the best harmony, the best proportions of the mouth in its relations to the other features require that there shall be the full complement of teeth and that each tooth shall be made to occupy its normal position . . . normal occlusion."

The result of this philosophy was that the dental arch was intentionally expanded in many cases. It was anticipated that the supporting alveolar bone would functionally sense the necessity of the change, and reposition itself to adequately support the bodies of all the teeth. John Hunter[4], 1778, held the belief that the loss of teeth could actually handicap bone growth in the jaw area.

As the problems of retention and stability began to haunt the profession, the importance of a full complement of teeth within an arch form began to yield to criteria such as the maintenance of teeth over their apical base. Excess tooth material was eliminated, and environmental factors[2] were heeded in estimating the position which the teeth, in function would seek.

In the development of arch form, hereditary factors are unquestionably involved. In each individual, the arch form which was established in the primary dentition can continually be altered by a variety of factors including eruption of teeth, change from deciduous to permanent dentition, muscle imbalances, tooth migration, skeletal growth, appliance therapy, etc.

There were many qualitative terms which described the varied arch forms found[5].

> "A tapered arch converges from the molars to the central incisors to such an extent that lines passing through the central grooves of molars and premolars intersect a short distance anterior to the central incisors. A trapezoid arch converges in variable degrees from the molars to canines. The anterior teeth are somewhat squared to abruptly rounded from canine tip to canine tip. Ovoid arches curve continuously from the molars on one side to the molars of the opposite side in such a way that two such arches placed back to back describe an oval. U-shaped arches present little difference in diameter between the first premolars and the diameter between the last molars, and the curve from canine to canine is abrupt so that capital "U" is formed."

Other terms such as paraboloid[3], catenary[6], hyperboloid and squared[5], etc. have been used to describe dental arches.

More recently, with the age of computers and more sophisticated scientific technology, dental arch form has been described more quantitatively. The traditional linear measurements such as arch width, arch length, arch depth, etc. have given valuable guidelines as far as which dimensions can reasonably be altered in a denture, or which dimensional changes can be expected in the transition from the deciduous to the permanent dentition. However, the actual form of the entire dental arch is a considerably more involved phenomenon which can only be quantified objectively by mathematical functions. In an investigation completed in June 1979, Coombs and Deming[7] concluded that:

1) Dental arch form can be closely approximated by a mathematical model.
2) Arch form is changed significantly during orthodontic treatment.
3) Change in arch form produced by orthodontic treatment is not stable. Changes in arch form continue once retention is discontinued. The direction of post-treatment change, at least in the upper arch, is back towards the form of the original malocclusion.
4) The lower arch form appears to be more amenable to permanent change from the original form than the upper, though both arch forms changed significantly after treatment.
5) Treatment and post-retention changes in intercanine and intermolar widths in the sample were generally consistent with previous studies of orthodontically treated cases.

The purpose of the present study was to evaluate if arch form changes significantly during maturation in untreated cases in an age group approximately the same as that in the study of Coombs and Deming. In this way, the present study was intended to serve as a bona-fide control for evaluating, authenticating, or modifying the conclusions of Coombs and Deming.

REVIEW OF THE LITERATURE

John Hunter[4] was one of the first to describe age changes in arch form. In 1771 he wrote, "The jaw increases in all points till twelve months after birth, when bodies of all the six teeth are pretty well formed; but it never after increases in length between the symphysis and the sixth tooth; and from this time too, the alveolar process, which makes the anterior part of the arches of both jaws, never becomes a section of a larger circle . . . never increases in length between the symphysis and sixth tooth." Later in his book Hunter wrote, "As that part of each jaw which holds the ten fore teeth is exactly the same size when it contains those of the first set as when it contains those of the second, and as these last often occupy a much larger space than the first, in such cases the second set are obliged to stand very irregularly."

In the twentieth century, various authors reported naturally occurring dimensional changes in the dental arch. Angle, in his Treatment of Malocclusion of the Teeth[3], wrote, "The permanent teeth being larger and more numerous than the deciduous, the greater space required by them is provided by the broadening of the dental arches in the region between the canines, and the lengthening of their lateral halves, posterior to the deciduous molars." Lewis and Lehman[8] in 1929 stated that the deciduous arches increased in size to allow for the increased size of the permanent incisors. They found that the greatest increase was in the canine region, and after the age of six years. In 1936, Lewis[9] reported on a period of "accelerated growth" in the intercuspid area during the eruption of the incisors, and also a second period of accelerated growth beginning with the eruption of the permanent cuspids. He considered that the essential factor in good alignment and occlusion of the permanent teeth of the anterior segment of the arches was not the occurrence of deciduous incisor spacing before the eruption of the permanent incisors, but "growth adjustments during or after eruption." He suggested that the alveolus modified itself in adapting to the changes occurring to tooth position. The adaptation, whether it be to treatment changes during this period or natural changes, was implied to be necessary for stability and good occlusion of the denture. Brash[10], 1928, found that the characteristic form of the deciduous arch is changed to that of the permanent arch "by growth in width of the posterior part of the arch, which takes place during the tooth change, and by the non-vertical succession of the anterior teeth."

Goldstein and Stanton[11], in 1935, published an extensive study of the effects of growth upon form. In primarily cross-sectional fashion, they studied 300 children between the ages of 1 and 11 years with both normal and abnormal occlusion. None of the subjects had had orthodontic treatment except for posterior deciduous space maintainers in about 20 percent of the sample. Their results showed that the maxillary and especially the mandibular arches became relatively broader between 2 and 9 years. The mandibular arch had a proportionately greater reduction in absolute length than the maxillary arch.

In a longitudinal study, Cohen[12] in 1940 examined 15 boys and 13 girls from ages 3½ to 13½ years, and made dimensional measurements from cusp tips. He found that the distance between the lower cuspids reached a maximum at 8½ years, whereas the upper cuspids continued to widen until the age of about 12 years. In the deciduous molar region there appeared to be no growth, and the space in fact was "shorter at 13½ years than it was at 3½ years of age".

Another longitudinal study of human lower dental arches by Speck[13], 1950, agreed with the findings of Brash[10] that the posterior segments usually expanded. Speck reported also that the anterior segments of most lower arches became flatter, "a segment of a larger circle." His sample covered the transition from the complete deciduous to permanent dentition. Richardson and Brodie[14], in a longitudinal study of maxillary width, concluded that the apical base of the maxilla, anterior to the first permanent molars, "becomes shorter and wider, the arc of a larger circle."

In 1951 Brown and Daugaard-Jensen[15] reported on cast measurements for 24 non-treated individuals studied from their early teens to early twenties. The range of the ages was 12 years, 10 months to 21 years, 6 months. Their results showed an average decrease in both intercanine width and intermolar width for both dental arches during the specified time period. The average arch width decrease approached 1 mm in all categories. Arch length was found to decrease in both arches by an average of approximately 1.5 mm. Another observation was that there was a tendency to space closure and an increase in crowding with age.

Barrow and White[5], 1952, in a longitudinal study of 51 children, found that maxillary and mandibular dental arches changed only a little from the primary to the early permanent dentition. In general, the changes "consisted of an increase in tapered and trapezoidal arches and a decrease in ovoidal ones." The intercanine width increased about 4 mm in the maxilla and 3 mm in the mandible between the ages of 5 to 9 years, and then decreased 0.5 mm to 1.5 mm after the age of 14. In the permanent first molar area, the width increased an average of 1.8 mm in the maxilla and 1.2 mm in the mandible between the ages of 7 to 11 years, and then decreased 0.4 mm and 0.9 mm, respectively, from 11 to 15 years. From 15 to 17 years, more than one-half of their cases showed a continued decrease in intermolar width.

Arch length measurements showed an increase of 1.0 mm in the maxilla and a decrease of 1.12 mm in the mandible during the period 6 to 12 years. From 12 to 13½ years, there was a decrease in the dental arches of 0.5 mm and 0.67 mm, respectively. Cumulatively, in the period from 4½ to 13½ years, the total change of length was a 0.2 mm increase in the maxilla and a 2.2 mm decrease in the mandible. In many of their cases, the length of the dental arches continued to decrease through 17 or 18 years of age.

Brodie[16], 1953, gave credit to muscle forces for their influence in guiding the eruption of teeth and consequently the original shape of the dental arches. He felt that late changes in muscle forces due to tension, habits, etc. could make significant changes in the shape of the dental arches. Lavelle[17], in a study of age changes in the dental arch shape, stated that shape may vary with such factors as muscle pressure.

Vargervik[18], in a report on morphologic evidence of muscle influence on dental arch width, came to a similar conclusion.

In 1961 Knott[19] reported on 29 white children who had been studied for a period of at least six years, and obtained differing results from those of Barrow and White[5], 1952. She used a different landmark to measure width (base of the first molar rather than the occlusal surface) and included only subjects with acceptable occlusion, but found increases in intermolar width over the same 11 to 15 year period. In the maxilla, the increases were 1.2 mm and 0.4 mm for boys and girls, respectively. The corresponding increases in the mandible were 0.9 mm and 0.4 mm. From ages 9.5 to 13 years, Knott found intermolar arch width increases of 1.4 mm for boys (both arches) and 1.2 mm (both arches) for girls.

Comparable age groups studied by Moorrees[20] produced smaller intermolar width increases than those reported by Knott. Corresponding values for intermolar width increase reported by Woods[21], in a longitudinal radiographic study, were also smaller.

In comparing her data on arch depth changes with that of Brown and Daugaard-Jensen, Knott found that most of the arch depth decrease which occurred in the Brown and Daugaard-Jensen sample up into the early twenties could be accounted for in her group by age 15 years. The amount of the arch depth decrease approximated 1.5 mm for boys and girls in the upper arch, and 2.0 mm in the lower. Moorrees[20] showed much smaller (less than 1.0 mm) decreases through the ages 9 to 14 years.

Sillman[22], 1964, in a longitudinal study of 65 normal white children, reported decreases in arch length during the period 3 to 25 years of 1.5 mm in the maxilla and 2.0 mm in the mandible. Dental arch width did not change significantly in males after age 14 and in females, after age 16. His group consisted of what he wanted to be a "random sample" of the population, and therefore included good and poor occlusions. Cleft palate and cleft lip patients were excluded from the study, but patients who had had prior orthodontic treatment were not excluded.

Lavelle and Foster[23], 1969, in a cross-sectional study of 1020 English subjects, found that the dimensions of the dental arches generally increased up to nine years of age in the anterior, and up to eleven to thirteen years in the other regions of the jaws, "there being little change thereafter."

In 1970 Lavelle[24] used a canonical analysis to evaluate data from the casts of 280 subjects ages 3 to 15. He found that the maximum changes in the human dental arch occurred in the two periods between 5 to 7 years and 11 to 13 years. These changes correspond to the period of change from the deciduous to the permanent dentition. In another publication, Lavelle[17] compared the arch index (a ratio of arch width divided by arch length) of different age groups and found it to increase maximally between the ages of 5 to 7 and 11 to 13. A 1975 study by Lavelle[25] of Caucasoids, Mongoloids, and Negroids showed similar age changes in arch index irrespective of race.

DeKock[26], 1972, conducted a longitudinal study of 26 subjects with good occlusion from 12 years to early adulthood. He found that for each one of his subjects arch depth continued to decrease throughout the period from 12 to 26 years. For females, the arch depth decrease from 15 to 26 years was found to be nearly equal to arch depth decrease from 12 to 15 years. For males the decrease from 15 to 26 years was found to be only 2% less than the decrease which occurred from 12 to 15 years. Males also showed a small statistically significant increase in arch width from 12 to 15 years of age.

Knott[27], 1972, found that in most instances the maximum bicanine diameter was reached in both arches when the stage of permanent dentition was attained. This corresponded to a mean age of 13.6 years, and little dimensional change occurred after that point. The pattern of change in the second premolar area was very similar.

In 1975 Lavelle[28] found that mandibular arch width and skull dimensions at the second molars were greatest in the Class III and least in the Class II samples. The maxillary arch width at the second molars was found to be greatest in the Class II group and least in the Class III. Thus different arch forms should be expected which are characteristic of the type of occlusion or malocclusion which they represented.

The studies which have been cited have primarily dealt with dimensional changes in the dental arches. However, it is quite clear that arch width dimensions do not change in exact proportion to arch depth or length dimensions. Therefore, changes in overall dental arch shape were occurring. For the most part, these changes could be described as a broadening of the dental arches, because the length dimension consistently showed a decrease which is greater than that shown by arch width.

It is apparent from a review of the literature that the majority of arch form changes occur in the transition from the mixed to the permanent dentition. Significant changes occur in the early mixed dentition period as well. There is, however, some disagreement as to whether or not significant change occurs after the permanent dentition, from the first molars forward, is attained.

It is in this permanent dentition period when the greater portion of active edgewise orthodontic treatment is undertaken. Coombs and Deming[7] have reported that arch form was changed significantly by orthodontic treatment. They obtained data from the study models of 30 individuals at three periods of time: 1) just prior to the start of orthodontic treatment, 2) at the completion of orthodontic treatment, and 3) several years out of retention. Their study was one of only a few which have attempted to fit mathematical equations to less than "ideal" occlusions, and their results indicated that a spline curve equation could be generated that would accurately fit even a malocclusion.

The use of spline curves had been suggested by Pepe[29] in a 1975 paper which evaluated the fit of catenary and fourth and sixth order polynomial equations to the dentitions of seven children with normal occlusion. Other authors are cited by Coombs and Deming[7] in their thorough and accurate review of "historical descriptions of arch form."

Bobkin[30] recently used the cubic spline function in the analysis of growth and treatment changes in dental arch form. He used a variety of knot points which were individualized to give "best fit" to the arch form presented by the pre and post-treatment study models of 30 patients. His results suggested that combinations of knot points could produce a spline curve representative of any patient's arch form, even if teeth were malaligned. A matched-pair t-test was performed to compare pre and post-treatment maxillary and mandibular dental arches. The perpendicular distances from the spline curve, or "normals", to the actual anatomical landmark were used for the evaluation of fit. It was found that statistically significant differences existed between the average normals of pre and post-treatment maxillary and mandibular arches. It was also noted that in a fairly high percentage of cases with large pretreatment sum or average normals, extractions were performed during the course of treatment. Bobkin[30] thus suggested that the splines could be used in treatment planning, building treatment objectives into arch wires, and evaluation of growth and treatment changes in arch form.

Mathematical descriptions of dental arch form have greatly aided this area of research. The computer is limited, of course, by the accuracy of the data entered, but its ability to objectively quantify the minute changes occurring in relation to a complex mathematical model is unprecedented. Computer analysis and mathematical models are making possible the comparison and interchange of data which heretofore was totally impractical.

MATERIALS AND METHODS

The materials for this study consisted of study models of 32 cases from the Child Study Clinic at the School of Dentistry, University of Oregon Health Sciences Center, Portland, Oregon. The criteria for case selection were: 1) without regard to presence or absence of malocclusion, 2) no form of orthodontic treatment had occurred, and 3) study models were available for the early permanent dentition and at a time six or more years later.

In each case, two sets of study models from the longitudinal cast series were selected. The first set of models examined was the earliest one with a fully erupted permanent dentition from the first permanent molars forward (designated Time 1). The second set of models was taken after a "maturation" period of at least six years had elapsed (Time 2). Casts showing gross dental abnormalities, missing teeth, etc. were eliminated from the sample along with the cases which were not in sufficiently good condition to permit precise anatomic markings.

The sample (Appendix A) consisted of 16 males and 16 females. The breakdown of the sample according to Angle's classification was: Class I - 18, Class II division 1 - 2, Class II division 2 subdivision - 1, and 1 Class III subdivision. In addition, there were 4 cases which began with an end to end molar relationship which resolved to a Class I relationship by the end of the study period. There were 5 cases which at Time 1 were end to end or Class II on one side and Class I on the other side, which remained unchanged through the maturation period. One case began as end to end bilaterally and changed to Class I on the right side, the left remaining unchanged. The mean age at Time 1 was 13 yrs. 3 mos. with a range from 11 yrs. 0 mos. to 16 yrs. 0 mos. The mean age at the end of the maturation period was 23 yrs. 5 mos. with a range of 18 yrs. 0 mos. to 31 yrs. 0 mos. The mean duration of the maturation period was 10 yrs. 2 mos. with a range 6 yrs. 7 mos. to 16 yrs. 5 mos.

In order to obtain standardized photographs of the study casts from which the data could be recorded, an orientation procedure was followed. The procedure was identical in most respects to that used by Coombs and Deming[7] and the references to Figures 1, 3, 4, and 5 used below are from their report. The casts were oriented on surveyor tables and dental and soft tissue landmarks were identified with ink and photographed in the following manner:

First the lower cast was placed on a surveyor table which was adjusted so that the occlusal plane of the cast was parallel to the base of the surveyor table. The occlusal plane was defined as the plane formed by the distobuccal cusp tip of both lower first molars and the incisal edge of the most anterior lower incisor, modified after the method used by Moyers, van der Linden, Riolo, and McNamara[31]. The upper cast was then placed in occlusion on the lower cast held in the surveyor table. Three horizontal cast orientation marks were scribed on the base of the upper cast, two on either side of the heel and one on the front of the base, all three scribe marks being the same distance from the base of the surveyor table (Figure 1[7]). Corresponding marks were made on the upper and lower casts by noting and marking one centric occlusion point on each side of the lower cast which transferred to the upper when the casts were pressed together while the water soluble ink was still wet. These marks were subsequently used to transfer the Y axis from the upper cast to the lower cast.

The upper cast was then mounted on a surveyor table and the three horizontal cast marks were oriented equidistant from the base of the surveyor table. With both casts on surveyor tables and the defined occlusal plane parallel to the surveyor table bases, the anatomic landmarks were marked with water soluble ink (Sanford's Vis-a-Vis, black). The soft tissue landmarks (Figure 3[7]) used in this study were: a) the most dorsal indication of the midpalatal raphe; and b) the most ventral point on the midpalatal raphe behind incisive papilla. The dental landmarks utilized were: a) the buccal cusp tips of the molars and premolars; b) the cusp tip of the canines; and c) the mid-point on the incisal edge of the incisors.

The casts were photographed with a 35 mm Nikkormat single lens reflex camera with a 100 mm lens and bellows using Kodak Plus-X Pan black and white film (ASA 125). To insure a fixed focal distance on all casts photographed and to facilitate standardized enlargement, an orientation table was constructed (Figure 4[7]). The camera was mounted on a tripod and kept at a constant distance above the orientation table. The casts mounted on surveyor tables were then raised on a laboratory jack through an aperature in the orientation table so that the occlusal plane was level with the surface of the orientation table.

Standardized enlargment of the photographic negatives was controlled through the use of a millimeter ruled graph paper scale on the surface of the orientation table. This millimeter scale was employed as a guide for the enlargement of all prints a uniform amount which was approximately 2½ times actual size to facilitate the digitizing procedure and minimize errors. Four fiducial marks were marked on the graph paper on the surface of the orientation table to form a rectangle 80 mm apart on the horizontal and 50 mm apart on the vertical axes. These fiducial marks were utilized for subsequent computer correction of all measurements to actual size.

X and Y axes were constructed on the enlarged photographs of the upper and lower casts for orientation during digitizing (Figure 5[7]). First, a line was drawn that passed through the midpalatal raphe marks on the upper cast. Next, a reference line was drawn on each photograph to connect the two centric occlusion marks. On the upper photograph the distance from the point of intersection of the Y axis with the reference line to the articulation mark on one side was measured. This distance was transferred to the reference line on the photograph of the lower arch by use of dividers. The angle of intersection of the Y axis and the reference line in the upper was measured with a protractor and transferred to the lower photograph and the Y axis was drawn in. Care was taken in transferring the angle of intersection and the point of intersection to the lower as they were in both cases on opposite sides in the two photographs. A line representing the X axis was drawn perpendicular to the Y axis in the upper and lower photographs so that it passed through the most anterior incisor midpoint in each arch.

The data points on teeth were all numbered for identification purposes. Numbering always started at the most posterior left tooth in the lower and most posterior right tooth in the upper. It should be noted that a specific number did not necessarily refer to the same tooth at all time periods of a given case, as they were numbered consecutively taking into account only those teeth present in the arch at that time, i.e., in the Time 1 models, for some cases, the second molars were not present while they were present in the succeeding models.

From the photographs, the dental and soft tissue points that were marked on the models were digitized at Oregon State University using a Calma Company Model 303 X, Y digitizer. The precision of this machine is listed at $\pm$.01 inch. The photographs were oriented in relation to the X, Y axes of the digitizer by positioning each so that the origin of the cross hairs of the digitizer overlaid the origin of the X, Y axes. As the digitizer scale was in inches, after the data was recorded on computer cards it was converted to millimeters and then reduced to the original scale by means of a computer program. All computer computations and statistical analyses were performed on a CDC CYBER 70/73 at Oregon State University.

After all data had been digitized, a program was used to generate the midpoint between the canine and adjacent premolar on each side of the arch to be used for a knot in the construction of the spline curves (Figure 3[7]). A knot is that point which is chosen to divide the arch into segments. A third knot (Figure 5[7]) in each arch was located at the origin (0, 0) of the digitized cartesian coordinate system. Each arch, therefore, had three knots connecting the four resulting curve segments.

STATISTICAL ANALYSIS

Third degree spline curves[32] were derived via a computer program using the method of least squares to obtain the coefficients that minimize the sum of squares $\sum_{i}^{n} (y_1 - \hat{y}_2)^2$ where y_1 is the Y coordinate of the data point and $\hat{y}_2$ is the value predicted by the polynomial. The spline curves fit to the arch were of the third order and of the following form:

$$\hat{y} = b_0 + b_1x + b_2x^2 + b_3x^3 + b_4(x-t_1)^3 + b_5(x-t_2)^3 + b_6(x-t_3)^3$$

where t_1, t_2, and t_3 are the X coordinates of the three knot points in each arch. All data points in the arch were used to estimate the coefficients b_0, b_1, b_2, and b_3. Points in the second, third, and fourth segments were used to estimate b_4; points in the third and fourth segments to estimate b_5; and only points in the fourth segment to estimate b_6. Because the equation is forced through the origin of the coordinate system (t_2), $b_0 = 0$ and is not a factor in the equation. For each arch, the coefficients of determination (R^2) and mean square error (MSE) were computed to evaluate how closely the points predicted by the spline curves fit the data points.

A second method of statistical evaluation was the fitting of the

Time 1 generated spline curve for each arch to the Time 2 data points. Differences in the distance between the predicted data points using the Time 1 coefficients and the actual data points at Time 2 were reported as MSE.

In order to compare the arch forms at each time, vectors of coefficients derived from the spline curve fitting procedure were examined by means of a multivariate analysis of variance using Wilk's lambda.

ERROR OF THE METHOD

The overall error of the method was determined from 4 randomly selected sets (Time 1 and Time 2) of upper and lower models, which were 12% of the sample. The dental and soft tissue points on the models were remarked, the models were oriented on the surveyor tables, and photographed. The axes were then constructed on the photographs and the photographs digitized as had been done originally.

The error of the method was evaluated by comparing the difference in MSE generated by fitting the coefficients of the cubic spline of the original data points to the replicate points of the same arch. The following formula was utilized:

$$\sum_{\substack{\text{all} \\ \text{patients}}} \frac{MSE_1 - MSE_2}{\text{\# of patients}}$$

where MSE_1 is the mean square error from the original data and MSE_2 is the mean square error of the replicated points.

To determine how much of the total error was due to errors in the digitizing procedure, 4 upper sets and 4 lower sets of the original marked photographs were submitted for redigitization. The digitizing error was evaluated in the same way as the total method error.

The error of the method is derived from a combination of the procedures preparatory to generating the spline curves. The most likely sources include error in location of the data points marked on the teeth and soft tissues, error in the construction of the axes used in aligning the photographs for digitizing, and error in the digitizing procedure.

FINDINGS

The findings of this study are presented in Tables I-IV. Tables I-A and I-B list the ocefficients of determination (R^2) and mean square error (MSE) of the cubic splines fit to the data points of each study model. For most of the cases, MSE was less than 10 mm^2 for both Times 1 and 2 in both upper and lower arches.

The fairly low MSE values for most of the sample indicate that the predicted points fit the observed data points quite closely. The average MSE for the upper arch was 4.255 mm^2 at Time 1 and 3.672 mm^2 at Time 2. The MSE increased in 16 and decreased in 15 of the upper arches from Time 1 to Time 2. The MSE increase from Time 1 to Time 2 ranged from 0.007 mm^2 to 4.137 mm^2 with an average of 1.27 mm^2. The MSE decrease from Time 1 to Time 2 ranged from 0.006 mm^2 to 8.49 mm^2 with an average of 3.32 mm^2.

In the lower arch the average MSE was 2.673 mm^2 at Time 1 and 3.157 mm^2 at Time 2. The MSE values increased in 19 and decreased in 12 of the subjects from Time 1 to Time 2. The MSE increase from Time 1 to Time 2 ranged from 0.020 mm^2 to 10.854 mm^2 with an average of 1.626 mm^2. The MSE decrease in the lower arch ranged from 0.012 mm^2 to 4.937 mm^2 with an average of 1.263 mm^2.

The R^2 values were high for all cases at both time periods, which indicated that a very high percentage of the variation of the dental arch forms were accounted for by the spline curve fits. The mean R^2 for the upper arch was .986 at Time 1 and .990 at Time 2. In the lower arch the mean R^2 was .990 for both Time 1 and Time 2. In the upper arch, R^2 increased in 17 cases from Time 1 to Time 2. The increase in the upper arch ranged from 0.001 to 0.035 with an average of 0.010. R^2 decreased in 10 of the cases with the decrease ranging from 0.002 to 0.016 and an average of 0.004. Four of the upper arch cases showed no change in R^2 from Time 1 to Time 2.

In the lower arch the R^2 showed an increase in 13 cases which ranged from 0.001 to 0.015. The average increase was 0.004. R^2 remained unchanged in 3 of the lower arch cases, and decreased in 15. The decrease ranged from 0.001 to 0.010, with an average of 0.004.

The results from the upper arch of case #123.4 and from the lower arch of case #252 were omitted for the study due to obvious errors in the digitizing procedure. Thus, 31 upper and 31 lower arches were evaluated, although 32 subjects were involved.

Table II shows the MSE values obtained when fitting the cubic spline equation for the Time 1 data to the Time 2 data points. In only one case, #267 upper arch, did the MSE decrease when fitting the Time 1 equation to the Time 2 data. The average MSE for Time 1 in the upper arch was 4.255 mm^2 but for Time 2 this value was 20.264 mm^2 using the Time 1 generated spline. For the lower arch the mean MSE was 2.673 mm^2 at Time 1 and 21.201 mm^2 for Time 2. For the Time 1 data, the MSE ranged from 0.514 mm^2 to 14.019 mm^2 in the upper arch and from 0.176 mm^2 to 13.894 mm^2 in the lower arch. However, the variability of the MSE increased dramatically in the Time 2 data, the values ranging from as low as 1.913 mm^2 to as high as 107.49 mm^2 in the lower arch. The range for Time 2 in the upper arch was from 2.652 mm^2 to 75.486 mm^2.

Table III shows the multivariate analysis of variance of the vectors of the coefficients of the spline curves compared at both time periods. The Wilk's lambda was 0.43589 in the upper arch and 0.52733 in the lower arch. The corresponding F values were 3.6 and 3.35. The Wilk's lambdas and their associated computer calculated F values were found to be significant for time comparisons of both upper and lower arches ($p = .01$).

The results of the procedure to determine the error of the method are tabulated in Table IV-A. For the upper arch the total method error as indicated by MSE was 3.499 mm^2 and in the lower arch the total method error is 5.287 mm^2. These differences are due to an accumulation of errors involved in marking, orienting, and photographing the models, as well as marking and digitizing the photographs.

The digitizing error has been separated out from the error of the method in Table IV-B. The resolution of the digitizer was 0.01 inch (0.254 mm). The redigitizing of the same photographs resulted in an average MSE increase of 0.693 mm^2 in the upper arch and 0.922 mm^2 in the lower arch.

DISCUSSION

Coombs and Deming[7] demonstrated that cubic spline curves could be generated that accurately fit even the dental arches of less than ideal alignment. According to their results, the spline curves fit maloccluded arches nearly as well as Pepe's[29] fourth degree polynomial equation fit well aligned "ideal" arches. Pepe[29] suggested spline curves might improve the accuracy of the fit of sixth degree polynomials. Bobkin[30] showed that various combinations of knot points could be used to adequately fit a cubic spline to malaligned arches. His method, however, enlisted the cusp tips and incisal edges as knot points for the curve in most cases. In this way the mathematical curve was often forced to pass through points which were substantially out of alignment. The resultant skewed curve then did not appear to be an accurate representation of the actual arch form.

Bobkin[30] found a significant difference between pre and post-treatment arch forms using a matched-pair t-test. The t-test assumes a normal distribution in the sample, and may not be appropriate for a reliable interpretation of results.

In the Coombs and Deming[7] study, and again in the present study, the number of knot points was limited to three. The statistical analysis utilized coefficients of determination (R^2) and mean square error (MSE). The low number of knot points gave a greater number of degrees of freedom which were important for the reliability of the results.

The mean square error is the sum of the square of the distance (in mms) each observed data point is from that point predicted by the spline, divided by the degrees of freedom (in this case n-6, 6 being the number of coefficients). The root mean square (RMSE) is the square root of the mean square error, and is an estimation of the average linear distance each data point is from the spline curve. The average RMSE for Times 1 and 2 were 2.063 mm and 1.916 mm, respectively, for the upper arch, and 1.635 mm and 1.777 mm, respectively, for the lower arch.

Coombs and Deming[7] found that at Time 1 (their untreated state) the RMSE was 2.53 mm in the upper arch and 2.46 mm in the lower arch. At Time 2, immediately following active orthodontic treatment, RMSE was 1.647 mm for the upper arch and 1.209 mm for the lower arch. One must be cautious, however, in comparing these values to the values from this study due to the difference in sample. The present study involved a population of orthodontically untreated patients, yet the reason for non-treatment may have been that treatment was not indicated. The patients were all given dental exams at regular intervals, and if orthodontic treatment was indicated the patient was so advised. On the other hand, the subjects involved in the Coombs and Deming[7] study were about to embark in fully banded appliance therapy, and therefore their occlusions warranted significant intervention. It is likely that their sample contained a higher percentage of cases with malalignment and rotation problems. This may be a possible explanation for why their spline curve fits at Time 1 were not as good as those in the present study. At Time 2 their fits were better due to good alignment as a result of orthodontic treatment.

During the "maturation" period in this study, the maxillary fits were found to improve while the mandibular fits were noted to worsen on the average. No explanation is offered for the improved fit in the upper arch at Time 2, but the tendency to crowding and decreased arch length in lower arches with age[15,26,33] might account for their poorer fit at Time 2.

In this study the error of the fit of spline curves, as indicated by the MSE, was greater in the upper than in the lower arch. At Time 2 this difference became much less pronounced. Nonethless, the finding of better fit with mandibular arches was consistent with the findings of Coombs and Deming[7], using maloccluded and treated arches, and also Pepe[29], using a sample of "good" arches. It may be that the more uniform size of the mandibular anterior teeth, the relative size of the arch, or some other factors make the lower arch more amenable to mathematical curve fitting than the upper. This apparently is not just a characteristic of spline functions, as Pepe's[29] results utilized a sixth degree polynomial. Hechter[34] found the parabola to have a very high "goodness of fit" to both upper and lower arches, but did not state that either upper or lower arch was superior in fit compared to the other. His data excluded landmarks from teeth posterior to the first permanent molar.

The mean R^2 values indicate the amount of variation accounted for by the splines. It is expected that as the MSE decreases, the R^2 values will increase, and vice versa. The high R^2 values obtained in this study indicate that very little variation in each individual curve was unexplained by the curve, and supported the validity of the low MSE results.

Of major concern to orthodontists is the period of postretention change in the treated dentition. This change is unpredictable and a cause of much disappointment for patient and practitioner alike. Many of the papers cited in the review of the literature conclude that there is little arch form change in untreated cases after the attainment of the permanent dentition. DeKock[26] found evidence of continuing arch depth decrease after the age of 15 years, and even after the age of 17 years. He stated that it may be possible that relapse problems, such as the return of crowding in the lower incisor area, may be at least partially related to decrease in arch depth after these ages. For females, DeKock[26] found that arch depth decrease from 15 to 26 years of age is nearly equal to arch depth decrease from 12 to 15 years. For males, the amount of decrease during the ages 15-26 years was only 2% less than the arch depth decrease from 12-15 years. Every person in his group of 26 subjects, all with acceptable occlusion, showed a decrease in arch depth from 15-26 years of age. DeKock[26] also found a slight increase in intermolar width in males between the ages of 12 and 15 years.

Barrow and White[5] also gave evidence for continuing dental arch length decrease through 17 or 18 years of age. Their findings demonstrated, in general, "That the permanent teeth through the years move and wear in many ways resulting in a shortening of the dental arches."

Barrow and White also found a decrease in the lower intermolar width of 0.7 mm to occur in non-treated patients between the ages of 11 and 15 years.

Although the studies of DeKock[26] and Barrow and White[5] involved changes in certain arch dimensions, it is most likely that changes in arch form accompanied the dimensional changes. The results in the present study support this supposition. In comparing the mean square errors generated by fitting a cubic spline to Time 1 data, and then fitting the same equation to the Time 2 data, it is apparent that the MSE increased significantly. The average MSE for the upper arch increased from 4.255 mm^2 at Time 1 to 20.264 mm^2 at Time 2, a difference of 16.009 mm^2 (RMSE 4.00 mm). The average MSE for the lower arch increased from 2.673 mm^2 to 21.201 mm^2 in the same period, a difference of 18.528 mm^2 (RMSE 4.304 mm). These differences were small in comparison to the changes which occurred in the treated arches of the Coombs and Deming[7] study.

An unpublished investigation by Kanarek[35] fitted a cubic spline to Time 1 arches of Coombs and Deming[7], and then fitted the same equation to their Time 2 arches. The MSE was found to increase from 6.407 mm^2 (RMSE 2.53 mm) to 110.006 mm^2 (RMSE 10.488 mm) in the upper arches and from 6.029 mm^2 (RMSE 2.46 mm) to 65.450 mm^2 (RMSE 8.09 mm) in the lower arches for the time interval specified. These values indicate that the spline curve fit was much less accurate at Time 2 than Time 1 suggesting a greater change in arch form from Time 1 to Time 2 than in the present study. Due to the length of the maturation period, a part of the maturational change may be related to some of the postretention change seen in the Coombs and Deming patients.

In the present study a portion of the difference between Time 1 and Time 2 MSE can be attributed to method error. None of the difference was due to the fit of the cubic spline as the same equation was used to obtain the MSE for both times. The difference in MSE therefore, was due to a combination of the total method error and the actural difference in the dental arches.

An estimate of the method error was found to be RMSE 1.87 mm (MSE 3.499 mm^2) in the upper arch and RMSE 2.30 mm (MSE 5.2871 mm^2) in the lower arch. Therefore, in the upper arch RMSE 2.13 mm (RMSE 4.00 - 1.87 mm) of the difference in fit to the original mathematical model at Time 2 can be ascribed to actual change in the dental arches during the maturation period. For the lower arch, the actual change in dental arches amounts to RMSE 2.00 mm (RMSE 4.30 - 2.30 mm) over the same time period.

The digitizing error, which is a portion of the total method error, amounted to MSE 0.693 mm^2 (RMSE .832 mm) in the upper arch, and MSE .992 mm^2 (RMSE .996 mm) in the lower arch. The original cubic spline equation was used for both Time 1 and Time 2 data, therefore none of the digitizing error was due to inaccuracies of the mathematical model. Subtracting a digitizing error from total method error yields error due to marking of the dental and soft tissue points on the models, orientation of the models, photography of the models, and marking of the photographs. The resultant method error was RMSE 1.038 mm in the upper arch and RMSE 1.304 mm in the lower arch.

A second method was used to compare the fit of the Time 1 and Time 2 splines by comparing the vectors of the coefficients. Using a multivariate analysis of variance, Coombs and Deming[7] obtained a Wilk's lambda value of 0.359 in the upper arch, and 0.478 in the lower arch between Time 1 and Time 2. The corresponding F statistics were 7.129 and 4.369, which were highly statistically significant ($p = 0.002$ and $p = 0.004$, respectively). Wilk's lambda values range from 0.0 to 1.0, with 0.0 being very significant and 1.0 insignificant. The Wilk's lambda values obtained in the present study were 0.43589 for the upper arch and 0.52733 in the lower arch, and the corresponding F statistics were 3.6 and 3.35 ($p = 0.01$ for both). These values are statistically significant, but minimally so in comparison to the values obtained in the Coombs and Deming study. Thus age changes in the dental arches of young adults are statistically significant, but not nearly as significant as the changes in the dental arches of a similar population which has undergone fully banded edgewise orthodontic treatment. Nevertheless, the present findings modify the assumption by Coombs and Deming that maturational changes in the dental arches are insignificant.

When evaluating the usefulness of this study, one must take into account the content of the sample population. Some of the drawbacks of the sample include: 1) it does not accurately represent a population which is about to undergo orthodontic treatment, 2) the cases had varying length of maturation periods, 3) the maturation period, which averaged 10 years 2 months, was considerably longer than the period of time most orthodontic patients would be undergoing active treatment, and 4) it consisted of only 32 individuals. A larger and less variable population might yield more meaningful results.

SUMMARY AND CONCLUSIONS

This study consisted of an evaluation of the dental arch form from study models of 32 individuals at two time periods from a longitudinal cast series: 1) the earliest one with a fully erupted permanent dentition from the first permanent molars forward, designated Time 1, and 2) after a "maturation" period of at least 6 years had elapsed, designated Time 2. Casts showing gross dental abnormalities, missing teeth, etc. were eliminated from the sample along with the cases which were not in sufficiently good condition to permit precise anatomic markings. The mean age at Time 1 was 13 years 3 months, and at Time 2, 23 years 5 months. The average duration of the maturation period was 10 years 2 months. The sample was chosen without regard to type or severity of malocclusion, but nevertheless consisted of primarily Class I cases.

The casts were related to each other by contact markings transferred from one cast to the other when the casts were pressed together in centric occlusion. The occlusal planes were made parallel to the base of a surveyor table, and the anatomic landmarks on each of the cases were marked. The landmarks chosen were the buccal cusp tips of the posterior teeth, the cusp tips of the canines, and the middle of the incisal edges of the anterior teeth. The most dorsal and ventral extensions of the midpalatal raphe were also marked on the upper casts.

Each cast was photographed individually and enlarged 2½ times. X and Y axes were constructed on the upper photographs by use of the midpalatal raphe marks for the Y axis, and a perpendicular to the Y axis through the most anterior incisor midpoint for the X axis. These axes were then transferred to the mandibular arch by use of the articulation marks.

Each of the data points on the photographs were digitized to obtain their X and Y coordinates which were scaled back down to actual size by use of fiducial points. By use of a computer program, a cubic spline curve was fit to the data points using the method of least squares.

Coefficients of determination and mean square errors were computed independently at each time period to evaluate how closely the predicted points derived from the spline curves fit the data points. The Time 1 spline equations were also used to evaluate Time 2 data points. A comparison of the vectors of the coefficients of the splines at each time period was done using a multivariate analysis of variance.

The total method error was evaluated by comparing the difference in the mean square errors generated by the spline curves fit to the original data points, and those generated by fitting the same equation to the replicated data points. Digitizing error, which accounts for a portion of the total method error, was evaluated separately, also by applying the original spline equation to the redigitized data. By using only the one spline equation in the error analysis, the actual difference between the Time 1 dental arches and Time 2 dental arches did not include any error due to inaccuracies of the mathematical model.

This project was undertaken as a control for authenticating or modifying the conclusions of the Coombs and Deming[7] study which utilized spline curves to evaluate dental arch form during and following orthodontic treatment. The age group was approximately the same and the dental casts were marked, oriented, photographed, and digitized in very similar fashion.

The conclusion by Coombs and Deming[7], that dental arch form can be closely approximated by a mathematical model, was substantiated in the present study.

The following conclusions can be drawn from this study:

1) Dental arch form can be closely approximated by a mathematical model.

2) Arch form can change a statistically significant ($p = 0.01$) amount without orthodontic treatment.

3) Arch form changes during a maturational period are small in comparison to changes produced by orthodontic treatment.

A future study might investigate if there are any differences in maturational changes between subjects with "good" occlusions and subjects with malocclusions that should undergo treatment. However, a population with malocclusion that should be treated, accompanied by adequate records, would be very difficult to come by.

BIBLIOGRAPHY

1. Tweed, C. H. "A Philosophy of Orthodontic Treatment," Am. J. Orthod. and Oral Surg., 31:74-103, 1945.

2. Strang, R. H. "Factors of Influence in Producing a Stable Result in Treatment of Malocclusions," Am. J. Orthod. and Oral Surg., 32:313-332, 1946.

3. Angle, E. H. Treatment of Malocclusion of the Teeth, S. S. White Dental Mfg. Co., Philadelphia, Pa., 1907.

4. Hunter, J. The Natural History of the Human Teeth, 2nd Ed., London, 1771.

5. Barrow, G. V. and White, J. R. "Developmental Changes of the Maxillary and Mandibular Dental Arches," Angle Orthod., 22:41-46, 1952.

6. Scott, J. H. "The Shape of the Dental Arches," J. Dent. Res., 36:996-1001, 1957.

7. Coombs, C. L. and Deming, D. C. "An Evaluation of Dental Arch From During and Following Orthodontic Treatment as Determined by Cubic Spline Curves," Certificate Thesis, University of Oregon Health Sciences Center, Dept. of Orthodontics, June, 1979.

8. Lewis, S. J. and Lehamn, I. A. "Observation of Growth Changes of the Teeth and Dental Arches," Dental Cosmos, 71:480-499, 1929.

9. Lewis, S. J. "Some Aspects of Dental Arch Growth," J. Am. Dent. Assoc., 23:277, 1936.

10. Brash, J. C. "The Growth of the Alveolar Bone and Its Relation to the Movements of the Teeth, Including Eruption," Int. J. Orthod. and Oral Surg., 14:196-223, 1928.

11. Goldstein, M. S. and Stanton, F. L. "Changes in Dimensions and Form of the Dental Arches with Age," Int. J. Orthodontia, 21:357-380, 1935.

12. Cohen, J. T. "Growth and Development of the Dental Arches in Children," J. Am. Dent. Assoc., 27:1250-1260, 1940.

13. Speck, N. T. "A Longitidinal Study of Development Changes in Human Lower Dental Arches," Angle Orthod., 20:215-228, 1950.

14. Richardson, E. R. and Brodie, A. G. "Longitudinal Study of Growth of Maxillary Width," Angle Orthod., 34:1-15, 1964.

15. Brown, V. P. and Daugaard-Jensen, I. "Changes in the Dentition from the Early Teens to the Early Twenties," Acta Odontol. Scand., 9:177-192, 1951.

16. Brodie, A. G. "Muscular Factors in the Diagnosis and Treatment of Malocclusions," Angle Orthod., 23:71-77, 1953.

17. Lavelle, C. L. B. "Age Changes in the Dental Arch Shape," J. Dent. Res., 49:1517-1521, 1970.

18. Vargervik, K. "Morphologic Evidence of Muscle Influence On Dental Arch Width," Am. J. Orthod., 76(1):21-28, 1979.

19. Knott, V. B. "Size and Form of the Dental Arches in Children with Good Occlusion Studied Longitudinally from Age 9 Years to Late Adolescence," Am. J. Phys. Anthropol., 19:263-284, 1961.

20. Moorrees, C. F. A. The Dentition of the Growing Child, Harvard Univ. Press, Cambridge, 1959.

21. Woods, G. A. "Changes in Width Dimensions Between Certain Teeth and Facial Points During Human Growth," Am. J. Orthod., 36:676-699, 1950.

22. Sillman, J. H. "Dimensional Changes of the Dental Arches: Longitudinal Study from Birth to 25 Years," Am. J. Orthod., 50:824-841, 1964.

23. Lavelle, C. L. B. and Foster, T. D. "A Cross-Sectional Study Into Age Changes of the Human Dental Arch," Arch. Oral Biol., 14:71-86, 1969.

24. Lavelle, C. L. B. et al. "A Multivariate Analysis of Age Changes of the Human Dental Arch," J. Anat., 106:167-168, 1970.

25. Lavelle, C. L. "The Shape of the Dental Arch," Am. J. Orthod., 67:176-184, 1975.

26. DeKock, W. H. "Dental Arch Depth and Width Studied Longitudinally from 12 Years of Age to Adulthood," Am. J. Orthod., 62:55-66, 1972.

27. Knott, V. B. "Longitudinal Study of Dental Arch Widths at Four Stages of Dentition, Angle Orthod., 42:387-394, 1972.

28. Lavelle, C. L. B. "Dental and Other Bodily Dimensions in Different Orthodontic Categories," Angle Orthod., 45:65-71, 1975.

29. Pepe, S. H. "Polynomial and Catenary Curve Fits to Human Dental Arches," J. Dent. Res., 54:1124-1132, 1975.

30. Bobkin, A. D. "Use of the Cubic Spline Function in Analysis of Growth and Treatment Changes in Dental Arch Form," Master of Science in Orthodontics Thesis, Univ. of Illinois at the Medical Center Graduate College, 1979.

31. Moyers, R. E. et al. "Standards of Human Occlusal Development," Univ. of Michigan, Ann Arbor, 1976, pp. 5-26.

32. Smith P. L. "Splines as a Useful and Convenient Statistical Tool," Am. Statistician, 33:57-62, 1979.

33. Bjork, A. and Skieller, V. "Facial Development and Tooth Eruption," Am. J. Orthod., 62:339, 1972.

34. Hechter, F. J. "Symmetry and Dental Arch Form of Orthodontically Treated Patients," Journal of the Canadian Dental Association, 44(4):173-184, 1978.

35. Kanarek, P. Personal Communication, 1980.

TABLE I-A

Coefficients of determination (R^2) and mean square error (MSE)* of upper arch splines generated independently at Time 1 and Time 2

	TIME 1			TIME 2		
Case #	N (# points)	R^2	MSE	N (# points)	R^2	MSE
001	14	.993	1.412	18	.995	1.477
027	18	.974	10.084	18	.994	1.797
030	18	.988	4.010	18	.991	3.049
039	18	.964	14.019	18	.983	5.529
047	18	.978	5.985	18	.986	3.770
051	14	.996	.514	18	.993	2.037
056	18	.993	2.852	18	.988	5.249
066	18	.975	9.148	18	.994	2.355
082	18	.998	.760	18	.996	1.382
088	17	.996	1.824	18	.994	2.651
099	18	.991	2.466	18	.992	2.460
102	14	.994	1.374	18	.978	5.531
107	18	.981	5.882	18	.998	1.038
128	16	.996	1.541	18	.994	2.402
151	14	.994	1.725	18	.994	2.155
154	18	.990	3.425	18	.997	1.230
174	18	.979	7.217	18	.992	2.985

* (mm^2)

TABLE I-A (cont.)

Coefficients of determination (R^2) and mean square error (MSE)* of upper arch splines generated independently at Time 1 and Time 2

	TIME 1			TIME 2		
Case #	N (# points)	R^2	MSE	N (# points)	R^2	MSE
179	18	.981	5.556	18	.987	3.266
193	18	.988	3.881	18	.988	4.396
203	18	.978	8.394	18	.976	8.439
225	18	.999	.809	18	.999	.764
228	18	.985	6.097	18	.986	5.456
240	18	.990	4.109	18	.996	1.132
241.2	18	.997	1.028	18	.991	3.582
252+	18	.947	1.835	18	.982	5.799
254	16	.988	4.139	18	.985	5.503
256	18	.996	1.454	18	.993	2.479
267	18	.975	9.033	18	.979	7.392
305	18	.985	3.527	18	.993	1.680
312	16	.998	.589	16	.998	.596
316	18	.981	5.720	17	.986	3.673
Mean R^2		.986			.990	
Mean MSE			4.255			3.672

+ The corresponding lower arch data for this subject was not used in this study.

TABLE I-B

Coefficients of determination (R^2) and mean square error (MSE)* of lower arch splines generated independently at Time 1 and Time 2

	TIME 1			TIME 2		
Case #	N (# points)	R^2	MSE	N (# points)	R^2	MSE
001	17	.999	.176	18	.996	1.346
027	18	.978	7.209	18	.993	2.272
030	18	.960	1.323	18	.961	12.177
039	18	.998	3.999	18	.990	2.686
047	18	.995	.439	18	.992	2.162
051	16	.990	1.917	18	.992	1.626
056	18	.994	2.813	18	.984	6.386
066	18	.997	.998	18	.995	1.826
082	18	.988	3.183	18	.990	2.504
088	18	.998	.995	18	.992	2.962
099	18	.996	.738	18	.996	1.142
102	15	.996	.636	18	.989	2.632
107	18	.994	1.640	18	.996	.983
123.4+	18	.998	.576	18	.997	.881
128	18	.991	2.508	18	.993	1.659
151	17	.995	1.606	18	.995	2.034
154	18	.993	2.084	18	.991	2.072
174	18	.991	3.377	18	.990	3.397

* (mm^2)

\+ The corresponding upper arch data for this subject was not used in this study.

TABLE I-B (cont.)

Coefficients of determination (R^2) and mean square error (MSE)* of lower arch splines generated independently at Time 1 and Time 2

	TIME 1			TIME 2		
Case #	N (# points)	R^2	MSE	N (# points)	R^2	MSE
179	18	.978	4.563	18	.975	5.902
193	18	.992	2.377	18	.995	1.535
203	18	.960	13.894	18	.956	15.410
225	16	.991	2.851	18	.992	3.520
228	18	.999	2.465	18	.998	2.087
240	18	.993	2.345	18	.999	1.220
241.2	18	.995	2.253	18	.991	3.369
254	18	.990	2.109	18	.998	.383
256	18	.992	2.018	18	.998	.697
267	18	.995	1.288	18	.995	1.523
305	17	.991	2.071	18	.997	.664
312	18	.993	1.615	18	.990	2.602
316	18	.972	6.799	18	.965	8.193
Mean R^2		.990			.990	
Mean MSE			2.673			3.1565

TABLE II-A

Mean square errors generated by fitting a cubic spline to Time 1 data and the same equation to Time 2 data

UPPER ARCH

Case #	Time 1 MSE*	Time 2 MSE*
001	1.412	11.222
027	10.084	48.563
030	4.010	24.823
039	14.019	75.486
047	5.985	16.149
051	0.514	11.598
056	2.852	18.198
066	9.148	18.388
082	0.760	2.652
088	1.824	4.435
099	2.466	8.715
102	1.374	11.248
107	5.882	9.689
128	1.541	7.732
151	1.725	8.146
154	3.425	24.169
174	7.217	7.934

* mm^2

TABLE II-A (cont.)

Mean square errors generated by fitting a cubic spline to Time 1 data and the same equation to Time 2 data

UPPER ARCH

Case #	Time 1 MSE*	Time 2 MSE*
179	5.556	12.839
193	3.881	27.093
203	8.394	14.620
225	0.809	5.821
228	6.097	7.646
240	4.109	6.618
241.2	1.028	10.448
252	1.835	81.960
254	4.139	13.748
256	1.454	14.430
267	9.033	8.822
305	3.527	39.142
312	0.589	34.971
316	5.720	40.885
Average MSE	4.255	20.264

TABLE II-B

Mean square errors generated by fitting a cubic spline to Time 1 data and the same equation to Time 2 data

LOWER ARCH

Case #	Time 1 MSE*	Time 2 MSE*
001	0.176	12.331
027	7.209	107.49
030	1.323	27.188
039	3.999	29.201
047	0.439	42.462
051	1.917	15.879
056	2.813	8.769
066	0.998	1.913
082	3.183	5.130
088	0.995	9.615
099	0.738	4.802
102	0.636	31.699
107	1.640	19.163
123.4	0.576	8.817
128	2.508	12.992
151	1.606	13.900
154	2.084	25.774

* mm^2

TABLE II-B (cont.)

Mean square errors generated by fitting a cubic spline to Time 1 data and the same equation to Time 2 data

LOWER ARCH

Case #	Time 1 MSE*	Time 2 MSE*
174	3.377	6.295
179	4.563	33.147
193	2.377	17.470
203	13.894	20.055
225	2.851	24.717
228	2.465	10.661
240	2.345	11.994
241.2	2.253	15.186
254	2.109	13.606
256	2.018	17.753
267	1.288	7.156
305	2.071	18.356
312	1.615	45.883
316	6.799	37.816
Average MSE	2.673	21.201

TABLE III

Comparison of the vectors of the coefficients of the cubic splines fit independently to Time 1 and Time 2 data using a multivariate analysis of variance

Upper Arch Splines

	Df.	T_1 vs T_2	P
Wilk's lambda		0.43589	
F	6, 25	3.6	.01

Lower Arch Splines

	Df.	T_1 vs T_2	P
Wilk's lambda		0.52733	
F	6, 25	3.35	.01

TABLE IV-A

Error of the Method
(Including digitizing error)

UPPER

ID	Time	Original MSE*	Second Set MSE*
051	1	0.514	3.230
051	2	2.037	12.752
056	1	2.852	3.076
056	2	5.249	4.557
174	1	7.217	7.538
174	2	2.985	5.498
179	1	5.556	8.600
179	2	3.266	12.419
Mean		3.710	7.209
Difference		3.499	

LOWER

ID	Time	Original MSE*	Second Set MSE*
051	1	1.917	21.551
051	2	1.626	2.976
056	1	2.813	5.021
056	2	6.386	12.544
174	1	3.377	4.140
174	2	3.397	4.381
179	1	4.563	8.811
179	2	5.902	12.854
Mean		3.748	9.035
Difference		5.2871	

* mm^2

TABLE IV-B

Digitizing Error

UPPER

ID	Time	Original MSE*	Second Set MSE*
027	1	10.084	11.384
027	2	1.797	3.294
030	1	4.010	4.612
030	2	3.049	3.599
047	1	5.985	6.931
047	2	3.770	3.533
099	1	2.466	2.579
099	2	2.460	3.230
Mean		4.203	4.895
Difference		0.6928	

LOWER

ID	Time	Original MSE*	Second Set MSE*
027	1	7.209	11.160
027	2	2.272	2.070
056	1	2.813	3.903
056	2	6.386	6.427
099	1	0.738	0.868
099	2	1.142	1.149
240	1	2.345	2.405
240	2	1.220	3.518
Mean		3.016	3.936
Difference		0.922	

* mm^2

Appendix A

No.	Sex	Classification Start	Classification End	Age* at Start	Age at End	Maturation Period
001	F	E/E	I	11-0	26-0	15-0
027	M	I	I	14-0	21-1	7-1
030	M	I	I	15-0	27-0	12-0
039	M	I	I	14-0	27-0	13-0
047	F	I	I	13-2	24-0	10-10
051	F	I	I	12-0	25-2	13-2
056	M	I	I	14-0	22-4	8-4
066	F	I	I	13-0	23-0	10-0
082	F	I	I	12-0	19-0	7-0
088	F	I-L** E/E-R+	I-L E/E-R	13-1	21-0	7-11
099	F	I	I	13-6	27-0	13-6
102	M	E/E	I	12-0	24-11	12-11
107	M	III-L I-R	III-L I-R	15-0	23-3	8-3
123.4	F	I	I	14-0	22-7	8-7
128	F	I	I	14-0	20-7	6-7
151	F	I	I	12-0	28-5	16-5
154	M	E/E-L I-R	E/E-L I-R	12-0	19-0	7-0
174	M	I	I	13-0	26-0	13-0
179	F	I-L E/E-R	I-L E/E-R	11-0	25-0	14-0

* (yr-mo)
** (Left)
\+ (Right)

Appendix A (Cont.)

No.	Sex	Classification Start	Classification End	Age at Start	Age at End	Maturation Period
193	M	I	I	12-2	20-1	7-11
203	F	I-L II_1-R	I-L II_1-R	14-11	23-4	8-5
225	F	II_1-L I-R	II_1-L I-R	13-11	21-10	7-11
228	M	II-2 subd.	II-2 subd.	16-0	31-0	15-0
240	M	E/E	I	14-0	25-0	11-0
241.2	F	I	I	11-1	18-0	6-11
252	F	I-L E/E-R	I	14-1	21-0	6-11
254	M	I	I	14-1	21-0	6-11
256	M	II-1	II-1	12-11	25-7	12-8
267	M	I	I	14-1	22-0	7-11
305	F	E/E	E/E-L I-R	11-0	25-0	14-0
312	M	I	I	14-0	21-2	7-2
316	M	II-1	II-1	14-0	21-0	7-0