AN EVALUATION OF DENTAL ARCH FORM DURING AND FOLLOWING ORTHODONTIC TREATMENT AS DETERMINED BY SPLINE CURVES

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TABLE OF CONTENTS

PAGE

INTRODUCTION 4

REVIEW OF THE LITERATURE 7

MATERIALS AND METHODS 21

FINDINGS 28

DISCUSSION 33

SUMMARY AND CONCLUSIONS 43

BIBLIOGRAPHY 46

TABLES I - VII

FIGURES 1 - 5

APPENDIX

INTRODUCTION

During the early period of orthodontics which was greatly influenced by Edward Angle the accepted concept was that irregularities of teeth were due to a problem of occlusion. It was thought that if all the teeth were aligned in the proper "line of occlusion" with all inclined planes correctly related, a stable result would occur. This generally led to expansion of the dental arches with the idea that this would encourage nature to do its part to provide the necessary support. Vigorous use of the masticatory apparatus would then retain the new arch form. Little thought was given to the original arch form of the malocclusion[1,2,3,4,].

Time soon showed that many, if not most, of the cases thus treated failed to remain stable. Nevertheless, those early teachings were so revered that it was difficult for many orthodontists to accept that the principles behind their treatment could be wrong. Extraction was presented as an alternative to expansion in treatment, but it did not become generally accepted until Tweed, in 1940, presented 100 consecutive cases, treated by extraction, which showed greatly improved stability[5]. As extraction became more acceptable, the trend was to look more at the arch form of the original malocclusion and to use it as a guide in treating cases. It became a major goal of treatment to keep the teeth over the apical base. The importance of studying the original malocclusion was expressed by Strang[3] in 1946,

> "Far too little attention is directed to the study of the original malocclusion as an index of the most practical method of correcting the deformity when viewed from the standpoint of establishing a permanently stabilized product. The teeth are in harmony with their basal bones and there is correlation between the various environmental tissues and parts. In other words, a deformed denture is the product of abnormal forces that have reached a balance and furthermore, it is endowed with sufficient basal support to resist displacement of its component parts under functional stress. Now these two conditions are the most essential and powerful stabilizers of denture form and individual tooth location, both in normal and in malocclucing dentures, yet . . . they have received little or no consideration from the viewpoint of planned treatment. It is my belief that this is one of the main reasons why relapses are so frequently encountered when retaining appliances are removed."

It soon became accepted that retaining the original arch form of the malocclusion would aid in the stability of the treated denture. The intercanine and intermolar widths of the mandibular arch were generally used as guides for determining the limits of width in correcting the deformity[3,6]. Extraction was deemed necessary when these widths could not be maintained with non-extraction treatment.

Orthodontics is now entering a period of pre-angulated and pre-torqued edgewise appliances, preformed arch wires, and greater use of auxiliary help. All of these factors may make it easier for a practitioner to get into the habit of treating a majority of his cases to the same arch form dictated by the appliance manufacturer instead of the patient's original malocclusion. This could lead to failure to incorporate balance and harmony with the musculature, and, therefore, lead to less long-term stability of our cases.

The purpose of this paper is to re-evaluate the importance of maintaining original arch form in the treatment of malocclusions. Arch form changes that occur during orthodontic treatment and the stability of those changes will be evaluated using a mathematical model to define the shape of the dental arch. The following questions will be investigated:

1) Can arch form be accurately defined by a mathematical model?

2) Is arch form changed significantly during orthodontic treatment?

3) If arch form is changed during treatment, is the change stable?

4) Is the lower arch less amenable to change than the upper arch?

5) Are arch form changes as determined by a mathematical model consistent with those indicated by intercanine and intermolar width measurements?

REVIEW OF THE LITERATURE

This review will look at 1) the historical descriptions of arch form, 2) arch form changes with age in untreated occlusions, and 3) arch form changes due to orthodontic treatment and subsequent relapse.

I - HISTORICAL DESCRIPTIONS OF ARCH FORM

The shape of human dental arches has been referred to as elliptical, parabolic, or a combination of the two[7,10]. Other terms that sometimes have been used are U-shaped, horseshoe-shaped, hyperbolic, tapered, trapezoid, squared, ovoid, catenary, etc.[8,11]. These terms are generally used in a qualitative reference to the form of the dental arches and have, therefore, failed in most cases to meet the need of the orthodontist to quantify comparisons of the dental arches and to provide a means of arch predetermination. Numerous methods of arch predetermination and arch quantification have been suggested throughout the years in order to meet those needs.

Most of the early methods of arch predetermination were based upon measurement of the mesial-distal widths of the teeth that made up the arch. One of the earliest was the Bonwill-Hawley triangle. This method was originally proposed by Bonwill for the arrangement of artificial teeth, and later modified by Hawley[12], for use in orthodontia. It was founded on the premise that the ideal arch should be based on an equilateral triangle. Bonwill established his triangle on the average width between the condyles which he found to be four inches. Hawley felt that orthodontists could not measure the distance between the condyles accurately and, therefore, chose to base his arch on the width of the front teeth. The anterior form of the arch was that of a circle, the radius of which was the width of the central and lateral incisors and the cuspid. The final form of the arch was established by the projection of two equilateral triangles which would vary according to the widths of the front teeth.

Angle[9] felt that a method such as Hawley's may be valuable in approximating the true line of occlusion but he did not believe that it could accurately locate it. He defined his line of occlusion as "the line with which, in form and position according to type, the teeth must be in harmony if in normal occlusion". He described this line as being more or less a parabolic curve which varied within limits of normal, according to the race, facial type, temperament, etc.

Williams[13], 1917, did not agree that the arch varies according to facial pattern; that is, a round face indicated a round arch or a narrow face a narrow arch, etc. He set out to disprove it by sending pictures of an individual along with a set of artificial teeth to ten of the leading practitioners of the time, asking them to arrange the teeth in an arch according to type. He received widely varying opinions which tended to indicate that many practitioners had their own concept of what an ideal arch should look like which had no relation to the facial type of the patient. His studies led him to believe that the front teeth should be arranged on a circle whose center is midway between the buccal grooves of the first molars. He also believed that the ratio of the widths between the first molars and the cuspids should be maintained at approximately 14 to 9.

McCoy[14], 1919, was in agreement with Williams that arch form was not dependent on facial type. He felt that careful observation would convince anyone to discard the theory that nature always produces teeth in harmony with face and features. He used as evidence several skulls which he claimed showed no relation of the forms of the teeth to the shapes or sizes of the skulls. Commenting on the methods of arch predetermination, he stated that use of any of the methods which were based on the amount of tooth substance contained within the arch, along with recognized anatomic principles, would render a greater service than the operator depending on his "Eagle Eye" to guide him on his way.

Hellman[2], 1919, studied arch forms and tooth dimensions of anthropoid apes. He found that the form of the arch has no relation to the size of the teeth. He concluded that the form of the human dental arch is likewise not dependent upon the size of the teeth constituting it, and that the mathematical method of dental arch determination is unsatisfactory.

Stanton[15], 1922, criticized Hawley's method of arch form predetermination, claiming that it yielded the same inflexible form varying in size with the size of the teeth. This led to the shortest arch also being the narrowest. He claimed that the range of normal arches crossed, such that the narrowest arch became the longest. He devised a method of determining the ideal form of the arches of each case studied by using a map of the malocclusion and an "occlusal graph", and then placing the teeth on charts in a manner to assure the best occlusion with the minimum tooth movement.

Izard[16], 1927, did not believe that a constant ratio existed between the sum of certain denture dimensions and dimensions of the arch. He felt that the dimensions of the dental arch were governed by the corresponding dimensions of the face. A constant ratio was found to exist between the width of the maxillary arch and width of the face, and between the length of the arch and depth of the face. He established the form of the maxillary arch as an ellipse, the large axis of which was determined by measuring the auriculo-incisal radius with a radiometer and the small axis by measuring the bizygomatic distance with a large compass and then substracting the thickness of the soft tissue.

Chuck[17], 1934, suggested using the Bonwill-Hawley arch as an aid in constructing a symmetrically formed alignment arch wire which could then be altered according to the type of the individual, while maintaining the symmetry of the arch wire.

MacConaill and Scher[18], 1949, defined the common occlusal line as passing along the buccal cusp tips and incisal edges in the lower arch, and along the central fossae of the molars, occlusal fissures of the premolars, and incisal fossae of the canines and incisors of the upper arch. They looked at 25 sets of models and found that a catenary could be aligned to this curve of common occlusion in so many cases that it could be taken as the ideal curve of common occlusion. They defined the catenary as the curve assumed by a fine chain of many links suspended by its ends and allowed to hang freely. They rationalized that the catenary is the simplest curve in which teeth can be arranged and that any other curve would require extraneous force to distort the arcade.

Scott[19], 1957, also described arch form as a catenary curve. He likened the teeth, being tied together by transeptal fibers, to the links in a chain. He felt that the tooth germs lie in the form of a catenary, the form of which is maintained by the dentition due to alveolar process growth which remains more or less equal in amount and constant in direction in all parts of the arch.

Most of the more recent attempts to describe arch form have dealt with the use of mathematical equations and computers. Hayashi[20], 1962, used anatomical landmarks along the buccal cusps and incisal edges to study the curve of the dental arch. He found that the arch fit very well to the equation $y = ax^n \pm e^{\alpha(x - \beta)}$. Hayashi assumed symmetry of the arches and, therefore, looked at only one side of the arch.

Lu[21], 1966, felt that Hayashi's method was too cumbersome. He suggested the use of orthogonal polynomials for fitting equations to arch form. The even-powered polynomials measured the symmetry of the arch and the odd-powered the asymmetry. He found that the fourth degree polynomial fit the arch form quite nicely.

Currier[22], 1969, used a generalized polynomial least squares curve-fitting program to compare the ellipse and parabola to 25 pairs of plotted dental arch curves. The ellipse was found to be a statistically significant better fit to both the maxillary and mandibular arches than the parabola when considering the curve passing through the buccal cusps and incisal edges.

Biggerstaff[23], 1970, used a computer program for oscilloscopic simulations for correcting, orthodontically, problems in occlusion. These computerized "set-ups" were done, however, with regard to esthetics instead of stability. Biggerstaff[24] also suggested the use of the computer to estimate three variations in dental arch form--the ellipse, the hyperbola, and the parabola.

Brader[25], 1972, developed typically average curves within the trifocal elliptical family that brackets the range of observed arch size. He felt that these curves served as clinically useful arch form guides. He related the dental arch form to intraoral forces according to the equation PR = C where P is the pressure, R is the radius of curvature of an elliptical curve at the pressure site, and C is a mathematical constant.

Pepe[26], 1975, fit polynomial and catenary equations to the dentitions of seven children with normal occlusion. She found that neither catenary nor polynomial curves fit the dental arch well enough to serve as a template for an arch wire. The catenary fit the arch form least accurately. She also found that the 6th degree polynomial equations afforded significant increase in accuracy of fit over the 4th degree, which had been suggested for use by Lu. She felt that the 6th degree polynomial had potential as clinical indicators of arch form and, perhaps, malocclusion. She suggested that spline curves may also be found to have a high degree of accuracy of fit.

Hechter[27], 1978, found that the parabola (second order polynomial) fit the curve of the maxillary and mandibular arch very well. He fit the equations to both a curve defined by landmarks on the buccal cusp tips and incisal edges and to a curve defined by landmarks on the facial and buccal surfaces of the teeth. He used only normal occlusions and no second or third molars were included in the study. He felt that it probably didn't matter if a parabola, catenary, or ellipse was used to define the "mean" curve of the arch as long as the curve evaluated only went from first molar to first molar.

White[28], 1978, compared arch forms derived from four basic designs--the Bonwill-Hawley, the Brader, the catenary, and the Rocky Mountain Data Systems computer derived formula. His subjective opinion of their fit to 24 untreated superior adult occlusions was: 1) the catenary design had a good fit for 27% of the arches while the other three varied from 8 to 12%; 2) the R.M.D.S. computer-derived arch yielded 92% moderately good fit with no poor fits; 3) the Bonwill-Hawley, Brader, and catenary curves had between 40 to 46% moderately good fits with 27 to 52% poor fits. The catenary and R.M.D.S. computer-derived arch forms were superior over-all to the Brader and Bonwill-Hawley designs. White suggested that the lack of fit was due to asymmetry of the arches. Only 6.25% of the arches examined were judged to be symmetrical with 56.25% moderately symmetrical. He found that the teeth apparently arranged themselves in an arch dictated mainly by the osseous bases of the jaws. He, therefore, recommended using the bases as guides to construct customized ideal arch forms.

Everett and Matthews[29], 1978, used 4th degree polynomial equations to study the genetic control of dental arch form. They found that the mandibular arch was under greater genetic control than the maxillary.

Much has been written throughout the years about the value of mathematical descriptions of arch form, and they seem to have found a use, at least in research, in this age of computers. A generally accepted mathematical determination of arch form has yet to be agreed upon. Presently, the sixth degree polynomial appears to have a superior fit. A more accurate fit, however, may possibly be obtained with spline curves.

II - ARCH FORM CHANGES WITH AGE IN UNTREATED OCCLUSIONS

One of the first to describe the change of arch form with age was John Hunter[30] in 1771. He said, "the jaw increases in all points till twelve months after birth, when bodies of all the six teeth are pretty well formed; but it never after increases in length between the symphysis and the sixth tooth; and from this time, too, the alveolar process, which makes the anterior part of the arches of both jaws, never becomes a section of a larger circle . . . never increases in length between the symphysis and sixth tooth".

Speck[31], 1925, studied photographs of 52 series of casts. He found that in the majority of cases the form of the dental arch changed during the transition from complete deciduous to permanent dentition becoming flatter and wider in front and wider in back.

Barrow and White[11], 1952, found that the maxillary and mandibular dental arches changed only a little during the period from the primary to the early permanent dentition. They looked at 528 sets of serial casts of 51 children. The intercanine width increased about 4 mm in the maxilla and 3 mm in the mandible between the ages of 5 and 9, and then decreased 0.5 mm to 1.5 mm after the age of 14. The molars increased in width 1.8 mm in the maxilla and 1.2 mm in the mandible between 7 and 11, and then decreased an average of 0.4 mm in the maxilla and 0.7 mm in the mandible between 11 and 15.

Howes[32], 1960, stated that the basal arch outline from mandibular first molar to mandibular first molar, possibly alters little, if any, after the age of 5 years as Hunter indicated earlier. The coronal arch form can, however, often be enlarged during the mixed dentition and transitional stages of development in selected cases.

Richardson and Brodie[33], 1964, found that the apical base of the maxilla, anterior to the first permanent molars usually becomes shorter and wider, that is, the arc of a larger circle, during growth. They based their study on x-rays of plaster models and warned that their results may have been in error due to the second x-ray representing a different level of "cut" as a result of growth in height of the alveolar process.

Sillman[34], 1964, studied 750 casts of 65 persons. He found that in males canine width increased to 13 years of age in the maxilla and to 12 years in the mandible. No amounts were given but his graphs would indicate that the increases were small. The molar width in males increased 0.2 to 0.5 mm per year from the deciduous to second molar stage. After 14 years of age there was no evidence of a significant change. In females there was no significant change in either canine or molar width after the age of 16. One possible criticism of Sillman's study was that both orthodontically treated and untreated cases were grouped together.

Lavelle[35], 1970, used a multivariate technique to study age changes of the dental arch. He looked at dental arch width and arch length as measurements between several various landmarks. Looking at these arch dimensions individually he determined that they mainly increased up to 9 years of age in the incisor region and up to 11 - 13 years of age in the other regions of the arch, thereafter to remain virtually constant. He then used a canonical analysis to consider age changes of the dental arches as a whole. This method indicated that both size and shape of dental arches change maximally during periods from 5 - 7 and 11 - 13 years of age. This corresponds to the major phases of permanent tooth eruption. A later study[36] showed that the dental arch area of three different ethnic groups showed growth spurts during these same age periods.

DeKock[37], 1972, in a study of 16 males and 10 females with acceptable occlusion found no significant change in molar arch width in females after age 12. Males showed a slight increase of 0.6 to 1.0 mm between the ages of 12 and 15.

Knott[38], 1972, found that for most individuals, the maximum bicanine diameters of both arches showed little change after the permanent dentition was attained.

Though there are varying opinions the general concept seems to be that there is little, if any, arch form change, as described by arch width, after the permanent dentition is established. Changes that occur in these dimensions during orthodontic treatment can, therefore, be assumed to be a result of the treatment in most cases.

III - ARCH FORM CHANGES DUE TO ORTHODONTIC TREATMENT

One of the reasons for the continued study of dental arch form lies in the fact that orthodontists are seeking a way of determining how best to treat their patients so as to achieve the best possible long term stable result. Steadman[39] felt that orthodontic movement and retention of teeth only produced lasting changes in those patients where forces acted upon the teeth in such a manner so as to support them in their newly acquired positions. He added that those forces may be due to growth and development of bony, muscular, and nervous tissues, combined with newly-acquired functional and emotional activity. "Orthodontic movement of teeth per se does not establish any tooth in its new position ultimately."

In 1948 Webster[40] concluded that when a patient presents with a permanent dentition, muscle balance must be maintained and stability brought about and that because of this very little expansion of arch length or width should be attempted.

Lewis[41] felt that the dental arch would be more stable in the posterior portion of the arch if the width and buccolingual axial inclination of the original malocclusion were used as a guide in treatment and that the curvature of the lower anterior teeth should in essence duplicate that of the original.

Various researchers have reported on differing types of arch form and resultant stability that can result from orthodontic treatment. Because of its simplicity, they have relied upon intercanine and intermolar measurements to express their findings concerning arch form.

Lagerstrom[42] reported that in Class II division 1 treatment with a headcap-type head gear as the only appliance, the narrow peaked arch form typical of this type of malocclusion gradually assumed a more rounded form as did the apical base of bone supporting it. In such a case the intercanine width frequently increases dramatically along with its apical base. Point A in such cases, according to Lagerstrom will frequently be found to remain stationary or even move posteriorly, thus decreasing the midline length of the apical base as it would be measured from the lateral film alone.

Steadman[39] in his study of records of 31 cases from the University of Minnesota taken before active treatment, at the conclusion of treatment, and at least one year after the cessation of retention found that all of those patients who had bicuspids extracted showed a decrease in the upper intermolar width. Only three patients without extractions showed a decrease in intermolar width while in all of the other cases without extractions, it either remained unchanged or increased. He found that the lower intermolar width changes were similar to those of the upper except for three extraction patients who showed no change of intermolar width from before treatment to the final model. As to changes in the upper and lower intercuspid distances, those patients where bicuspids were extracted presented after retention no discernible differences from the patients who had had no bicuspids extracted. Thus extraction of bicuspids tends to result in a decrease of the upper and lower intermolar distances but produces no significant difference in the upper and lower intercuspid distances after retention is ended.

Sheppe[43] noted that the arch form in those cases which were treated by first bicuspid extraction were shorter and narrower and that the anterior outline was changed from the original.

After studying mandibular dental casts of 80 nonextraction and extraction cases 10 years out of retention by comparing them with the pretreatment and posttreatment records, Shapiro[44] found that the mandibular intercanine width showed a strong tendency to return to the pretreatment dimensions in all groups except in Class II division 2 cases which seemed to maintain some intercanine expansion. He also found that the mandibular arch length decreased substantially in every group but that reduction was less in the Class II division 2 group. The intermolar width decreased more in the extraction than in the nonextraction group from pretreatment to post-treatment. Most of the treatment expansion of molars was maintained in nonextraction cases even though there was a tendency to return to the original dimension. In extraction cases the intermolar width decreased during treatment and continued to decrease during the posttreatment period.

Johnson[45] looked at 11 cases and found similar results. He found that the cuspid width is most likely to decrease after treatment although on occasion a slight increase can be maintained. The molar width is apt to decrease from the beginning of treatment through the postretention period. He went on to say that lower arch crowding may be due to multiple factors such as expanded cuspids, protrusive and labially inclined mandibular incisors, and late skeletal growth changes.

Gardner and Chaconas[46] studied 74 nonextraction and 29 first bicuspid extraction cases. They made five measurements: intercanine, inter-first premolar, inter-second premolar and inter-first molar widths, and incisor to molar distance on the mandibular arch pretreatment, posttreatment, and postretention casts. The postretention model being obtained a minimum of one year after all retaining devices were removed. They noted that the following changes in dimensions occurred: The intercanine width was expanded during treatment but had a strong tendency to return to or close to its original pretreatment width in both nonextraction and extraction cases. The inter-first premolar width showed the greatest treatment increase in width with only a minimal amount of postretention decrease. The second premolar width for nonextraction cases showed a decrease with treatment and a slight continued decrease postretention. The intermolar width of nonextraction cases showed a significant increase in width with treatment. The extraction cases showed a significant decrease with treatment. However, there were no changes in either extraction or nonextraction cases postretention. The incisor to molar distance decreased with treatment and had a slight tendency to continue to decrease post-retention.

Schulhof[47] in his study of buccal expansion of the mandibular dental arch found that those cases which he studied that had a final intercuspid width of less than 27 mm showed significantly less relapse than those cases that had a final intercuspid width of 28 mm or more. He felt that the point of contact between the cuspid and the first premolar was the key point of the arch in determining the arch width.

Litowitz[48] studied 15 Class I and 5 Class II cases and found that during the course of treatment the width of the arches as measured by intercanine, first bicuspid, and intermolar distances showed an increase. These increases ranged from 1 mm to 10 mm. He found that some of this width was lost subsequent to retention but that the loss of width was not complete and that there was a considerable difference in the percentage lost in the various cases. He showed that expansion between the first bicuspids showed the least relapse tendencies of any of the other teeth in the buccal segments.

Chadha, Bishara, and Potter[49] studied 30 cases treated by the extraction of four first bicuspids. They found that the mandibular intercanine width increased during treatment by an average of just under 1 mm but that in the postretention period one half of that gain was lost. The maxillary intercanine width was increased by 3 mm during treatment but less than 1 mm of that expansion was lost postretention.

Walter[50] studied 238 sets of maxillary and mandibular models of patients who had been orthodontically treated nonextraction. From this he concluded that the statement that the dental arch cannot be permanently widened or lengthened was incorrect. Nine years[51] later, however, in a subsequent paper he did suggest that general arch form should be a consideration in preventing relapse.

Riedel[52] in his review of the evidence to date concerning retention stated that, "Arch form, particularly in the mandibular arch, cannot be permanently altered by appliance therapy, therefore, treatment should be aimed at maintaining, in most instances, the arch form presented by the original malocclusion."

MATERIALS AND METHODS

The materials for this study consisted of study models of 21 cases treated in the orthodontic department of the University of Oregon Health Sciences Center and 9 cases from the orthodontic department of the University of Washington. The cases were treated by graduate students using fully banded standard (.022 x .028) edgewise appliances.

In each case, three sets of study models were examined. The first set of models was taken before orthodontic therapy was instituted (designated Time 1). The second set of models was taken after active orthodontic appliances were removed and retainers placed (Time 2). The third set of models was taken as long after retention had been discontinued as was possible (Time 3).

The sample (Appendix A) consisted of 10 males and 20 females. The breakdown of the sample according to Angle's calssification was: Class I - 11, Class II division 1 - 14, Class II division 2 - 4, and one pseudo-class III. The mean age at the start of treatment was 12 yrs. 5 mos. with a range from 8 yrs. 11 mos. to 17 yrs. 2 mos. The mean age at the end of treatment was 15 yrs. 3 mos. with a range of 10 yrs. 5 mos. to 19 yrs. 9 mos. The mean age of the postretention models was 25 yrs. 11 mos. with a range from 20 yrs. 0 mos. to 30 yrs. 7 mos. The mean posttreatment period was 10 yrs. 10 mos. with a range of 4 yrs. 3 mos. to 15 yrs. 9 mos. and the mean postretention period was 9 yrs. 2 mos. with a range from 4 yrs. 3 mos. to 14 yrs 4 mos. There were 3 nonextraction and 27 extraction cases.

In order to obtain standardized photographs of the study casts from which the data could be recorded, an orientation procedure was followed. The casts were oriented on surveyor tables and dental and soft tissue landmarks were identified with ink and photographed in the following manner:

First the lower cast was placed on a surveyor table which was adjusted so that the occlusal plane of the cast was parallel to the base of the surveyor table. The occlusal plane was defined as the plane formed by the distobuccal cusp tip of both lower first molars and the incisal edge of the most anterior lower incisor, modified after the method used by Moyers, van der Linden, Riolo, and McNamara[53]. The upper cast was then placed in occlusion on the lower cast held in the surveyor table. Three horizontal cast orientation marks were scribed on the base of the upper cast, two on either side of the heel and one on the front of the base, all three scribe marks being the same distance from the base of the surveyor table (Figure 1). Corresponding marks were made on the upper and lower casts using articulating paper and pressing the casts together in centric occlusion. These marks were subsequently used to transfer the Y axis from the upper cast to the lower cast (Figure 2).

The upper cast was then mounted on a surveyor table and the three horizontal cast marks were oriented equidistant from the base of the surveyor table. With both casts on surveyor tables and the defined occlusal plane parallel to the surveyor table bases, the anatomic landmarks were marked with water soluble ink (Sanford's Vis-a-Vis, black). The soft tissue landmarks (Figure 3) used in this study were: a) the most dorsal indication of the midpalatal raphe; b) the most ventral point on the midpalatal raphe; and c) the lateral terminations of the most anterior pair of rugae. The dental landmarks utilized were: a) the buccal cusp tips of the molars and premolars; b) the cusp tip of the canines; and c) the mid-point on the incisal edge of the incisors.

The casts were photographed with a 35 mm Nikkormat single lens reflex camera with a 100 mm lens and bellows using Kodak Panatomic X black and white film (ASA 32). To insure a fixed focal distance on all casts photographed and to facilitate standardized enlargement, an orientation table was constructed as shown in Figure 4. The camera was mounted on a tripod and kept at a constant distance above the orientation table. The casts mounted on surveyor tables were then raised on a laboratory jack through an aperture in the orientation table so that the occlusal plane was level with the surface of the orientation table.

Standardized enlargement of the photographic negatives was controlled through the use of a millimeter ruled graph paper scale on the surface of the orientation table. This millimeter scale was employed as a guide for the enlargement of all prints a uniform amount which was approximately 2½ times actual size to facilitate the digitizing procedure and minimize errors. Four fiducial marks were marked on the graph paper on the surface of the orientation table to form a rectangle 80 mm apart on the horizontal and 50 mm apart on the vertical axes. These fiducial marks were utilized for subsequent computer correction of all measurements to actual size.

X and Y axes were constructed on the enlarged photographs of the upper and lower casts for orientation during digitizing (Figure 5). First, a line was drawn that passed through the mid-palatal raphe marks on the upper cast. Next, a reference line was drawn on each photograph to connect the two marks that were made by using articulation paper. On the upper photograph the distance from the point of intersection of the Y axis with the reference line to the articulation mark on one side was measured. This distance was transferred to the reference line on the photograph of the lower arch by use of dividers. The angle of intersection of the Y axis and the reference line in the upper was measured with a protractor and transferred to the lower photograph and the Y axis was drawn in. Care was taken in transferring the angle of intersection and the point of intersection to the lower as they were in both cases on opposite sides in the two photographs (Figure 5). A line representing the X axis was drawn perpendicular to the Y axis in the upper and lower photographs so that it passed through the most anterior incisor midpoint in each arch.

The data points on the teeth were all numbered for identification purposes as were the lateral rugae marks. Numbering always started at the most posterior left tooth in the lower and most posterior right tooth in the upper. It should be noted that a specific number did not necessarily refer to the same tooth at all time periods of a given case, as they were numbered consecutively taking into account only those teeth present in the arch at that time, i.e., in the Time 1 models, for most cases, the second molars were not present while they were present in the succeeding models. In Time 2 and 3 models, the extraction of bicuspids at the beginning of treatment altered the number of a specific tooth in comparison to Time 1.

From the photographs the dental and soft tissue points that were marked on the models were digitized at Oregon State University using a Calma Company Model 303 X, Y digitizer. The precision of this machine is listed at $\pm$ .01 inch. The photographs were oriented in relation to the X, Y axes of the digitizer by positioning each so that the origin of the cross hairs of the digitizer overlaid the origin of the X, Y axes. As the digitizer scale was in inches, after the data was recorded on computer cards it was converted to millimeters and then reduced to the original scale by means of a computer program. All computer computations and statistical analyses were performed on a CDC CYBER 70/73 at Oregon State University.

After all data had been digitized, a program was used to generate the midpoint between the canine and adjacent premolar on each side of the arch to be used for a knot in the construction of the spline curves (Figure 3). A knot is that point which is chosen to divide the arch into segments. A third knot (Figure 5) in each arch was located at the origin (0, 0) of the digitized cartesian coordinate system. Each arch, therefore, had three knots connecting the four resulting curve segments.

STATISTICAL ANALYSIS

Third degree spline curves[54,55] were derived via a computer program using the method of least squares to obtain the coefficients that minimize the sum of squares $\sum_{i}^{n}(y_1 - \hat{y}_2)^2$ where y_1 is the Y coordinate of the data point and $\hat{y}_2$ is the value predicted by the polynomial. The spline curves fit to the arch were of the third order and of the following form: $\hat{y} = b_0 + b_1x + b_2x^2 + b_3x^3 + b_4(x-t_1)^3 + b_5(x-t_2)^3 + b_6(x-t_3)^3$ where t_1, t_2, and t_3 are the X coordinates of the three knot points in each arch. All data points in the arch were used to estimate the coefficients b_0, b_1, b_2, and b_3. Points in the second, third, and fourth segments were used to estimate b_4; points in the third and fourth segments to estimate b_5; and only points in the fourth segment to estimate b_6. Because the equation is forced through the origin of the coordinate system (t_2), $b_0 = 0$ and is not a factor in the equation. For each arch, the coefficients of determination (R^2) and mean square error (MSE) were computed to evaluate how closely the points predicted by the spline curves fit the data points.

In addition, intercanine width and intermolar width were obtained from the digitized data. Intercanine width was measured from canine cusp tip to cusp tip while intermolar width was measured from the mesiobuccal cusp tip on one first molar to the mesiobuccal cusp tip of the contralateral first molar. In the event that one of the data points needed to compute intercanine or intermolar widths was missing, that dimension was not computed.

In order to compare the arch forms at each time, vectors of coefficients derived from the spline curve fitting procedure were examined by means of a multivariate analysis of variance using Wilk's lambda. A multivariate analysis of variance[56] was also used to compare simultaneously the intercanine and intermolar widths at the three times in each arch.

Canonical correlations[56] were used to compare the best linear (weighted) combination of the spline coefficients to the best linear combination of intercanine and intermolar widths. The squared canonical correlation is a measure of the variation in arch form as quantified by the spline coefficients which could be explained by the combination of intercanine and intermolar widths.

The upper and lower intercanine and intermolar widths at the three times were compared in a univariate manner by analysis of variance[56].

Significant differences were identified by the use of the least significant difference test.

ERROR OF THE METHOD

The error of the method was determined from 10 randomly selected pairs of models. These models were remarked, oriented, and photographed. The axes were then constructed and the photogrpahs digitized as had been done originally.

The replicated coordinates of the data points were compared with the values obtained from the original data. The X and Y coordinates were compared separately using the following equations: $\frac{\Sigma(X_{i_1} - X_{i_2})^2}{2N}$ and $\frac{\Sigma(Y_{i_1} - Y_{i_2})^2}{2N}$ where X_{i_1} is the value of the X coordinate of the original data point and X_{i_2} is the replicate value. The same notation is used for the Y coordinate. The error is from a combination of the procedures preparatory to generating the the spline curves. The most likely sources include error in location of the data points marked on the teeth, error in the construction of the axes used in aligning the photographs for digitizing, and error in the digitizing procedure.

FINDINGS

The findings of this study are presented in Tables I through VII. Tables I-A and I-B list the coefficients of determination (R^2) and mean square error (MSE) of the spline curves for each set of study models. It is readily seen that R^2 is high for all of the cases and that MSE is less than 10 mm^2 for most of the cases at all three time periods and for both upper and lower arches. This resulted in a mean R^2 in the upper arch of .980 with a standard deviation of .014 for Time 1, .991 with a standard deviation .007 for Time 2, and .990 with a standard deviation of .008 for Time 3. In the lower arch the mean R^2 was .977 with a standard deviation of .017 for Time 1, .995 with a standard deviation of .005 for Time 2, and .994 with a standard deviation of .006 for Time 3.

The mean MSE for the upper arch was 6.407 with a standard deviation of 4.724 for Time 1, 2.713 with a standard deviation of 2.293 for Time 2, and 2.079 with a standard deviation of 1.456 for Time 3. For the lower arch the mean MSE was 6.029 with a standard deviation of 5.008 for Time 1, 1.463 with a standard deviation of 1.206 for Time 2, and 1.378 with a standard deviation of 1.330 for Time 3.

The high R^2 values indicate that a very high percentage of the variation of the dental arch forms was accounted for by the spline curve fits. The low MSE in most cases indicates that the predicted points fit the observed data points quite closely.

In both the upper and the lower arches the highest mean R^2 was at Time 2, followed closely by Time 3, and then by Time 1. The MSE for both arches was lowest in Time 3, followed closely by Time 2, and then Time 1, which was much higher. The mean MSE was lower in the lower arch for all three time periods, indicating that on the average the curve fits were slightly better in the lower arch than in the upper.

Table II shows the multivariate analysis of variance of the vectors of the coefficients of the spline curves compared at each of the time periods. The Wilk's lambdas and their associated computer-calculated F values were found to be highly significant (from $p = .00001$ to $p = .036$) for each of the time period comparisons except for that between Times 1 and 3 in the upper arch, which was not significant.

A multivariate analysis of variance of the combined intercanine and intermolar widths compared at the different time periods is presented in Tables III-A and III-B. The results indicate a significant difference existed between each time period for both arches, with Wilk's lambda being significant at p levels ranging from .00001 to .002.

Table IV shows the results of the canonical correlations between the best linear combinations of the coefficients of the splines and the best linear combinations of the intercanine and intermolar widths. All of these correlations show statistical significance at the $p = .01$ level. The correlations obtained were generally better in the lower arch than in the upper, as indicated by the higher R^2's. All of the correlations (R's) were above .88 except T3 in the upper arch.

In Tables V and VI the intercanine and intermolar widths at each time period are compared separately by a univariate analysis of variance.

Table V-A indicates that the critical value (0.724) was exceeded in the upper arch for the difference in the mean intercanine widths between Times 1 and 2, and between Times 1 and 3, but not between Times 2 and 3. For the lower arch, as indicated in Table V-B, the mean difference in intercanine widths exceed the critical value (0.490) between Times 1 and 2, and between Times 2 and 3, but not between Times 1 and 3.

As shown in Table VI-A the critical value of the mean difference in intermolar widths of the upper arch was exceeded only between Times 1 and 3. Table VI-B indicates that for the lower arch the critical value (0.867) was exceeded in all three of the comparisons between the different time periods for this same measurement. However, the value between Times 2 and 3 was very close to the critical value.

The mean square differences between the original and the replicate coordinates of the data points for each case are shown in Tables VII-A and VII-B. The discrepancy is due to an accumulation of the errors involved in the marking of the anatomical landmarks, the photographic technique, the construction of the X and Y axes, the precision of the digitizer, and the error of the digitizing procedure.

While attempting to determine the source of greatest error, it became questionable as to whether these values were an accurate representation of the magnitude of error involved in the generation of the spline curves. By superimposing tracings from original photographs onto their replicates, it was apparent that the greatest error was involved in the location of the mid-palatal raphe points used to construct the Y axis of the upper arch. Since this axis was subsequently transferred to the lower arch, it resulted in a shifting of the origin of the X, Y coordinate system in both arches. As the intersection at the origin of the coordinate system was used to align the arches on the digitizer, it caused a shifting of most of the X coordinates of the replicate data points in the same direction as the displacement of the origin. The Y coordinates on the side of the shift were displaced in a negative direction and those on the side away from the shift were displaced positively. This direction of displacement is along the curve itself such that the points would probably lie closer to the curve. The error should, therefore, be less than is indicated by an assessment of the discrepancy of the individual X and Y coordinates of each point.

A study of the photographs also indicated that the same amount of displacement of the origin did not occur between the different time periods of a case as between the original cases and their replicates. This was due to the manner in which the casts were marked. In most cases the same anatomical landmark on each of the series of casts from all three time periods was marked at the same time. This allowed for greater consistency in marking the position of the midpalatal raphe landmarks on those casts where it was not clearly identifiable. The replicate casts were marked some time after all marks had been removed and without any comparison to the other casts. It is, therefore, expected that the method provided for a more accurate reproduction of the X and Y axes between the time periods than was indicated by the replicate photographs. Even so, the necessity of taking special care in marking the midpalatal raphe landmarks consistently has been well demonstrated.

The error of the photographic technique is assumed to be small since any inconsistencies in enlargement are negated when the computer reduces the data to its original dimensions by the use of the fiducial points on each photograph.

The resolution of the digitizer was 0.01 in. (0.254 mm). This is not as precise as desired and may therefore have accounted for more error than anticipated. The enlargement of the photographs reduces this error. The actual error of the digitizing procedure was not calculated.

A method of error determination that may be more representative of the actual methodological error than the one used in this study would be to fit a spline curve to the data points of the original case and obtain its mean square error. The same curve could then be fit to the replicate data points for that case and new mean square error obtained. The two mean square errors could then be compared to evaluate how much change had occurred in relation to the fit of the curve to the two sets of data points.

DISCUSSION

As far as can be determined, the use of spline curves to describe the human dental arch form has not been previously reported in the dental literature, although Pepe[26] stated that she planned to fit spline curves to her data as a means of improving the fit she obtained with sixth degree polynomials.

White[28] suggested that one reason for the inaccuracies of the fit of dental arches to geometric formulae is the asymmetry of the arches. Others[27,57,58] have also confirmed that asymmetry exists in even "normal" or "ideal" occlusions. Theoretically, a spline curve should provide a good fit to even an asymmetrical arch because it divides the arch into sections and does not assume any symmetry. This study involved the use of third degree splines fit to the arch divided into four sections. Prenter[59] in a book on splines and variational methods, however, cautions on the use of splines that "splines (are) a fine approximating tool . . . (but) . . . are not a panacea for all problems of numerical approximation".

The mean square errors give an evaluation of how closely the spline curves fit the data points. The square root of the MSE gives the average difference in millimeters between the observed data points and the predicted points. The square roots of the mean MSE's for Times 2 and 3 range from 1.17 to 1.21 mm in the lower arch, and from 1.44 to 1.65 mm in the upper. For Time 1 it is 2.46 mm in the lower and 2.53 mm in the upper. This indicates that the average deviation of the anatomical landmarks from the fit curves was between 1.17 to 1.65 mm for T2 and T3, and about 2.5 mm for T1. These findings represent an adequate fit of the spline curves to the actual data points compared to reports from similar studies. A still better fit would be desirable.

It would be appropriate to fit polynomial or catenary equations to this same data in order to compare the accuracy of fit of each. This has not been done at this time, however. A comparison is, therefore, made between this data and that reported by Pepe[26], keeping in mind that the two studies involved different cases and different sample sizes. Since her study looked at "good" occlusions, the best comparison would be with the spline curve fits to the data of Time 2 (T2), upon the completion of orthodontic treatment. The cases at this point should most closely resemble those of an untreated "good" occlusion, except for the possible extraction of bicuspids.

Pepe found the sixth degree polynomial provided a better fit than the second or fourth degree polynomials, or the catenary. Its MSE's were compared to the MSE's from the spline curves in this study. The sixth degree polynomial for the upper arch showed an average mean square error (MSE) of 6.5998 with a standard deviation of 2.8699 and a range of 3.8359 to 15.7905 (as calculated by the authors). For the lower arch her data had an average MSE of 2.4929 with a standard deviation of 0.8422 and a range of 1.8314 to 3.7013. The spline curves for Time 2 yielded a mean MSE of 2.7126 with a standard deviation of 2.2928 and a range of 0.32 to 9.35 for the upper arch, and a mean MSE of 1.463 with a standard deviation of 1.206 and a range of 0.019 to 5.41 for the lower arch.

These results do not necessarily mean that a spline curve provides a more accurate expression of dental arch form than a sixth degree polynomial because of the different samples the curves were fit to. It does, however, tend to support the idea that spline curves can be closely fit to data points on a dental arch. More direct comparisons, using the same sample are necessary in order to determine the relative accuracies of fit of the equations.

This study was one of only a few which have attempted to fit mathematical equations to less than "ideal" occlusions. The results indicate that a spline curve equation can be generated that will accurately fit even a malocclusion. The mean MSE for Time 1 (the maloccluded state) was 6.407 for the upper arches and 6.029 for the lower arches. The accuracy of fit is less than that for Times 2 and 3 of the splines, but the mean MSE's are still lower than those obtained by Pepe's description of "good" arches with a catenary equation (upper mean MSE 12.1136; lower mean MSE 7.8291), and are comparable to those obtained by a fourth degree polynomial equation (upper mean MSE 7.1284; lower mean MSE 3.1253). The few cases in this study with high MSE's were, with some exceptions, found to be severely crowded cases; such as, lower lateral incisors positioned almost directly behind the central incisors.

It would be expected that data points from finished orthodontic cases would be easier to align along a curve than those in an untreated malocclusion. This seems to be verified by the fact that both Times 2 and 3 had higher mean R^2's and lower mean MSE's than Time 1 in both the upper and the lower arches. The significance of the similarities in mean R^2's and MSE's of Times 2 and 3 indicate that though changes in individual cases may have occurred, the accuracy with which the data points could be fit by a spline curve equation was little affected.

The results offer no indication as to why the lower arches were fit better than the upper as indicated by the lower mean MSE's. Pepe's results for the sixth degree polynomial also showed smaller MSE's in the lower arch than in the upper.

The multivariate analysis of variance of the vectors of the coefficients of the spline curves indicates that for both arches there was a statistically significant degree of change in the form of the dental arch during orthodontic treatment. This is shown by the significant Wilk's lambda and associated F values between Time 1 and Time 2. The significant values between Times 2 and 3 indicate that the arch form change obtained during treatment was not stable, and that the form of the arch continued to change after retention was removed. This concept is by no means new to the orthodontic community. Angle[9] in 1907, stated that "the best the orthodontist can do is to secure normal relations of the teeth and correct general form of the arch, leaving the finer adjustment ... to be worked out by Nature through her forces which must, in any event, finally triumph".

Posttreatment change is well established but little understood. It has generally been referred to through the years as orthodontic relapse. The term relapse is used because it is generally believed that the teeth tend to return towards their original pretreatment position. The finding that the coefficients of the upper splines showed significant change between Times 1 and 2, and between Times 2 and 3, but not between Times 1 and 3 would tend to support the concept of orthodontic relapse. It indicates that even though there was a significant change during orthodontic treatment, the subsequent posttreatment change was in a direction back towards the original form of the malocclusion to such an extent that the final form of the arch was no longer significantly different from that of the original.

In the lower arch the form of the arch at Time 3 was still significantly different from that at Time 1. This may indicate that if the arch form was changing after the cessation of retention it was not back to the original maloccluded arch form, but in another direction. This would tempt one to postulate that the upper arch is less amenable to change than the lower arch as far as arch form is concerned. This is contrary to the idea expressed by Everett and Matthews[29] that the upper arch is under less genetic control than the lower and, thus, more amenable to permanent change. It does, however, lead one to consider the possibliity that the form of the upper arch is determined by the facial musculature and that the lower arch is contained within the upper.

It would not be wise to contend that the upper arch was less amenable to change based on this one finding alone because no relative magnitudes of change between the upper and lower arches between Times 1 and 2 and between Times 2 and 3 were examined--it was merely determined that significant change did or did not occur. It must also be remembered that arch form change, as referred to here, is concerned with change in the mathematical equations used to describe dental arches. This may or may not be directly related to linear dimension changes, return of crowding, spacing, or rotations, etc. which are often referred to when investigators speak of permanency of changes in the arches.

Though not used in this study, another method would be to fit the spline curve equation to the data points of the original malocclusion, and then fit this same equation to the data points of the two subsequent periods. This could be used to evaluate if posttreatment changes result in a return to the form of the original malocclusion, and if one arch is more amenable to change than the other. The mean square errors could then be compared to determine if the arch form at T3 more closely fits the original equation than at T2. The differences in the mean square errors between Time 1 and Time 3 for the upper arch could be compared to those for the lower arch. This would result in a quantification of the differences in the arrangement of the data points between the three time periods.

Another question that was not evaluated in this study, but which would be interesting to investigate is whether the orthodontic cases showing the greatest arch form change during treatment also show the greatest arch form change after treatment. This could be studied in the same way by fitting a spline curve to the original malocclusion, and then fitting the same spline curve to the Time 2 and Time 3 data points. The one-third cases showing the greatest MSE change between Times 1 and 2 would be the cases showing the most arch form treatment change, and those showing the smallest MSE change the least. The amount of MSE change that these cases show between Times 2 and 3 could be evaluated.

When the intermolar and intercanine widths were combined and compared at the various time periods, the results closely paralleled those of the comparison between the vectors of the coefficients of the spline curves at the same time periods. The only difference was between Times 1 and 3 in the upper arch, where the change in the coefficients of the spline curves was not significant; whereas, the change in the intercanine and intermolar widths was. The canonical correlations between the best linear combination of the coefficients of the splines and the best linear combination of intercanine and intermolar widths indicate a relationship between arch form, as described by spline curves, and the measurements of the intercanine and intermolar widths considered together. All of the correlations were significant at the p = .01 level. The lowest correlation was R = 0.7453 for Time 3, which some might consider to be too low to be of clinical significance. The other correlations ranged between 0.8812 and 0.9384, all of which would be considered to have clinical significance. These results would tend to indicate that there are other factors to be considered in dental arch form than just the intercanine and intermolar widths, but that when these two measurements are combined they can, in most cases account for a high proportion of the variation in dental arch form as described by spline curves.

A comparison of the intercanine and intermolar widths taken independently at each of the time periods was also made. Neither the intercanine nor intermolar comparisons were in complete agreement with the changes indicated by the comparison of the coefficients of the spline curves, but the findings were in most cases in agreement with those of previous researchers investigating those measurements.

The mandibular intercanine width has been the most frequently studied dimension of those widths examined in this paper. With the possible exceptions of Walter[50,51] and Shapiro[44] (who noted that

Class II division 2 patients seemed to maintain some of the intercanine expansion) all of the other papers reviewed in this project came to the same conclusion as this data would indicate. The data showed that the mandibular intercanine width had a significant mean increase during treatment but that the subsequent relapse was so complete as to make the difference in width between Time 1 and Time 3 not statistically significant. In non-orthodontically treated patients the maximum bicanine diameters of both arches have been shown to have little change once the permanent dentition is attained[38].

The maxillary intercanine width in this study was found to have a mean increase from Time 1 to Time 2 but partially returned to the Time 1 dimension at Time 3. Statistically there wasn't a significant difference between Time 2 and Time 3 though there was between Time 1 and Time 2 and also between Time 1 and Time 3.

Lagerstrom[42] also found an increase in the maxillary intercanine width though his sample consisted of Class II division 1 patients that were treated by head gear only. He didn't follow these patients to see if the increase was stable however.

Chadha, Bishara, and Potter[49] found a mean treatment increase in maxillary intercanine width of 3 mm but less than 1 mm of that amount was lost postretention. In non-orthodontically treated patients, Sillman[34] found that the intercanine width of the maxilla could still be increasing until age 13.

Although the mean maxillary intermolar width decreased between each time period, only the change between Time 1 and Time 3 was significant. The narrowing of this width after retention was also noted by Steadman[39] in patients treated by extractions while he didn't find it to be true of the nonextraction patients in his sample. The explanation why the treatment decrease in intermolar width was not significant could be due to the fact that in some Class II cases (which made up the bulk of the sample) upper molars are prevented from moving mesially into a narrower portion of the arch by means of headgear wear. Consequently the mean change in that time period was not quite significant. Postretention decrease also was not of sufficient magnitude. However, the overall combined change resulted in a total mean decrease at Time 3 which was significantly different from Time 1. This could be due to some anchorage loss in some cases during treatment combined with postretention mesial drift as band spaces closed in addition to the so-called "anterior component of force".[60]

The results of this study show that mandibular intermolar width had decreased significantly between all of the time periods. The decrease in width between Time 1 and Time 2 may be attributed in extraction cases to the mesial movement of the lower molars into a more narrow portion of the arch in order to achieve a Class I molar relationship. The further decrease in width after retention may again be possibly attributed to closure of band spaces and the well-recognized tendency for arch length to decrease, possibly due to an anterior component of force. Steadman[39] found that the lower intermolar changes were similar to those in the upper except that in three nonextraction cases from his sample there was no decrease in width. Shapiro[44] found that the intermolar width decreased more in the extraction than in the nonextraction group from pretreatment to posttreatment. He noted that in extraction cases, the intermolar width decreased during treatment and continued to decrease during the postretention period. Gardner and Chaconas[46] however, found that the intermolar width increased in non-extraction cases.

Maturational changes have also been noted by others. Barrow and White[11] found a decrease in the lower intermolar width of .7 mm to occur in non-treated patients between the ages of 11 and 15. DeKock[37] found no change in intermolar width in females after 12 but that there was a slight increase in males between the ages of 12 and 15.

When evaluating the usefulness of this study, one must take into consideration the content of the sample. Some of the drawbacks of this sample include: 1) it consisted of all classes of malocclusions, 2) it included both extraction and nonextraction cases, 3) the cases had varying lengths of retention and postpretention periods, and 4) it consisted of only 30 individuals. A larger and less variable sample would yield more meaningful results. Despite the drawbacks, this study has shown that arch form changes can be evaluated by using a mathemetical expression of arch form.

It has been assumed in this paper that the changes seen during treatment and subsequent to treatment were exclusively a result of orthodontic treatment and postretention instability. It would be interesting to perform a similar study on a sample of untreated individuals over similar age periods to evaluate if any arch form changes might be due to maturation.

SUMMARY AND CONCLUSIONS

This study consisted of an evaluation of the dental arch form of study models of 30 individuals obtained at three periods of time: 1) just prior to the start of orthodontic treatment, 2) at the completion of orthodontic treatment, and 3) several years out of retention. The mean age at the start of treatment was 12 yrs. 5 mos.; at the end of treatment 15 yrs. 3 mos.; and, at the last set of models 25 yrs. 11 mos. The mean postretention period was 9 yrs. 2 mos. The sample consisted of mainly Class I and Class II malocclusions, except for one pseudo-class III. Both extraction and nonextraction cases were included.

The casts were related to each other in centric occlusion by the use of articulating paper. The occlusal planes were made parallel to the base of a surveyor table, and then anatomic landmarks on each of the casts were marked. The landmarks chosen were the buccal cusp tips of the posterior teeth, the cusp tips of the canines, and the middle of the incisal edges of the anterior teeth. The most dorsal and ventral extensions of the midpalatal raphe were also marked on the upper casts.

Each cast was photographed individually and enlarged 2½ times. X and Y axes were constructed on the upper photographs by use of the midpalatal raphe marks for the Y axis, and a perpendicular to the Y axis through the most anterior incisor midpoint for the X axis. These axes were then transferred to the mandibular arch by use of the articulation marks.

Each of the data points on the photographs were digitized to obtain their X and Y coordinates which were scaled back down to original size by use of fiducial points and converted to the millimeter scale. By use of a computer program a 3rd degree spline curve was fit to the data points, using the method of least squares.

Coefficients of determination and mean square errors were computed to evaluate how closely the predicted points derived from the spline curves fit the data points. The following statistical analyses were performed: 1) a comparison of the vectors of the coefficients of the splines at each time period using a multivariate analysis of variance; 2) a multivariate analysis of variance of the combined intercanine and intermolar widths compared between the three times; 3) canonical correlations between the best linear combinations of the coefficients of the splines and the best linear combination of intercanine and intermolar widths; 4) a univariate analysis of variance of intercanine width; and, 5) a univariate analysis of variance of intermolar width.

The following conclusions can be drawn from this study:

1) Dental arch form can be closely approximated by a mathematical model. Spline curves in this study deviated from the data points of the treated study models an average of 1.17 to 1.65 mm, and from the untreated models an average of 2.5 mm.

2) Arch form is changed significantly during orthodontic treatment.

3) Change in arch form produced by orthodontic treatment is not stable. Changes in arch form continue once retention is discontinued. The direction of posttreatment change, at least in the upper arch is back towards the form of the original malocclusion.

4) The lower arch form in this study appeared to be more amenable to permanent change from the original form than the upper, though both arch forms changed significantly after treatment.

5) The arch form changes determined by the spline curves were consistent with those indicated by the comparisons of intercanine and intermolar widths taken in unison, except for between start of treatment and several years out of retention in the upper arch.

6) Treatment and postretention changes in intercanine and intermolar widths in this sample were generally consistent with previous studies of orthodontically treated cases.

The following areas should be examined in future studies:

1) Comparison of the arches by fitting the spline curve equation obtained on the Time 1 data to the Times 2 and 3 data. The resulting MSE can then be compared.

2) In a similar manner those cases showing the most change during treatment can be compared to those cases showing the least treatment change, to see whether the postretention results follow the same pattern.

3) A study of untreated cases over similar age periods to evaluate if arch form changes during maturation.

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TABLE I-A

Coefficients of determination (R^2) and mean square error (MSE)* of upper arch splines

	TIME 1		TIME 2		TIME 3	
CASE NO.	R^2	MSE	R^2	MSE	R^2	MSE
403	.988	2.25	.995	1.22	.995	.713
406	.981	5.01	.996	1.05	.996	1.04
409	.990	3.65	.994	1.66	.994	1.74
412	.990	2.91	.990	2.99	.983	5.09
416	.991	3.15	.990	4.10	.993	1.81
418	.949	18.64	.990	2.97	.994	1.43
428	.994	1.94	.999	.32	.962	.96
483	.987	4.72	.981	5.29	.979	6.11
503	.986	4.30	.980	8.47	.987	4.03
534	.988	7.63	.999	1.79	.993	2.16
540	.972	6.70	.991	2.78	.992	2.77
554	.985	4.91	.989	2.89	.976	5.79
600	.974	8.63	.991	2.19	.998	.379
639	.997	.94	.997	.83	.996	1.03
660	.982	5.72	.989	3.36	.991	2.22
681	.979	6.41	.996	1.13	.996	1.01
720	.977	7.16	.998	.65	.997	.78
776	.957	12.78	.988	2.71	.990	2.04
818	.986	4.23	.993	1.95	.994	1.46

* (mm^2)

TABLE I-A (cont.)

Coefficients of determination (R^2) and mean square error (MSE) of upper arch splines

	TIME 1		TIME 2		TIME 3	
CASE NO.	R^2	MSE	R^2	MSE	R^2	MSE
820	.974	12.49	.996	1.12	.993	2.08
853	.992	2.58	.993	2.52	.991	2.95
9001	.997	1.32	.996	1.18	.994	1.56
9002	.965	9.73	.990	2.89	.994	1.92
9003	.984	4.32	.998	.56	.997	1.33
9004	.934	18.88	.967	9.35	.992	1.98
9005	.986	4.47	.996	1.12	.996	1.71
9006	.992	1.39	.973	7.77	.991	2.13
9007	.976	6.55	.996	1.05	.996	.99
9008	.961	12.39	.993	1.79	.996	1.07
9009	.977	8.09	.988	3.68	.988	3.39
Mean R^2	.980		.991		.990	
SD R^2	.014		.007		.008	
Mean MSE		6.407		2.713		2.079
SD MSE		4.724		2.293		1.456

TABLE I-B

Coefficients of determination (R^2) and mean square error (MSE)* of lower arch splines

	TIME 1		TIME 2		TIME 3	
CASE NO.	R^2	MSE	R^2	MSE	R^2	MSE
403	.933	18.53	.990	1.87	.999	.47
406	.986	2.67	.999	1.17	.997	.52
409	.978	6.80	.994	1.71	.995	1.68
412	.974	6.27	.998	.98	.997	1.67
416	.994	1.71	.992	2.56	.997	.82
418	.933	20.53	.996	1.31	.998	.44
428	.989	1.23	.999	.19	.998	.24
483	.994	1.82	.999	.37	.997	1.50
503	.958	8.95	.995	2.25	.994	2.21
534	.991	2.25	.995	1.46	.999	.32
540	.984	4.16	.999	.83	.993	1.67
554	.961	9.01	.991	1.57	.987	2.01
600	.996	1.65	.984	4.08	.997	.62
639	.968	10.73	.993	1.24	.996	.91
660	.972	8.71	.996	1.44	.983	4.24
681	.992	2.19	.999	.50	.994	1.47
720	.980	4.74	.999	.48	.995	1.21
776	.980	3.75	.996	.97	.991	1.86
818	.985	4.18	.997	.57	.995	1.02

* (mm^2)

TABLE I-B (cont.)

Coefficients of determination (R^2) and mean square error (MSE) of lower arch splines

	TIME 1		TIME 2		TIME 3	
CASE NO.	R^2	MSE	R^2	MSE	R^2	MSE
820	.993	1.83	.996	.79	.993	1.36
853	.994	2.17	.997	1.57	.996	.94
9001	.995	.91	.999	.26	.998	.34
9002	.963	7.78	.994	1.19	.995	1.01
9003	.976	6.02	.998	.69	.999	.68
9004	.973	7.12	.978	5.41	.997	1.22
9005	.981	4.50	.982	4.00	.970	6.95
9006	.986	1.63	.995	1.07	.999	.88
9007	.960	9.14	.998	1.00	.995	.83
9008	.954	13.87	.996	.91	.996	.89
9009	.982	6.10	.991	2.50	.991	2.36
Mean R^2	.977		.995		.994	
SD R^2	.017		.005		.006	
Mean MSE		6.029		1.463		1.378
SD MSE		5.008		1.206		1.330

TABLE II

Comparison of the Vectors of the Coefficients of the Third Degree Splines Using a Multivariate Analysis of Variance

<u>Upper Arch Splines</u>

	T1 vs T2 vs T3			T1 vs T2			T1 vs T3			T2 vs T3		
	Df.		p	Df.		p	Df.		p	Df.		p
Wilks lambda		0.690			0.359			0.818			0.594	
F	12, 106	1.834	∿ .05	6, 24	7.129	.0002	6, 24	0.889	0.5[NS]	6, 24	2.726	.036

<u>Lower Arch Splines</u>

	T1 vs T2 vs T3			T1 vs T2			T1 vs T3			T2 vs T3		
	Df.		p	Df.		p	Df.		p	Df.		p
Wilks lambda		0.361			0.478			0.291			0.417	
F	12, 106	5.878	.00001	6, 24	4.369	.004	6, 24	9.766	.0002	6, 24	5.587	.001

NS = Not Significant

TABLE III-A

Multivariate Analysis of Variance of
Intercanine and Intermolar Widths

Upper Arch

	T1 vs T2 vs T3	T1 vs T2	T1 vs T3	T2 vs T3
Wilks lambda	0.590	0.563	0.633	0.449
F	8.595	10.851	8.130	17.191
Df.	4, 114	2, 28	2, 28	2, 28
p	.00001	.00032	.002	.0001

TABLE III-B

Multivariate Analysis of Variance of Intercanine and Intermolar Widths

Lower Arch

	T1 vs T2 vs T3	T1 vs T2	T1 vs T3	T2 vs T3
Wilks Lambda	0.372	0.344	0.561	0.407
F	18.209	26.745	10.966	20.361
Df.	4, 114	2, 28	2, 28	2, 28
p	.00001	.00001	.0003	.00001

TABLE IV

Canonical Correlations

Correlations between the best linear combination of the coefficients of the splines and the best linear combination of intercanine and intermolar widths.

	T1		T2		T3	
	R	R^2	R	R^2	R	R^2
Upper Arch	0.8812	0.65810	0.9293	0.86358	0.7453	0.55547
Lower Arch	0.9322	0.86902	0.9384	0.88067	0.9219	0.84999

p = .01 for all correlations

TABLE V-A

Upper Arch

Comparison of intercanine widths at the three times by analysis of variance

Source	Df.	S.S.	M.S.	F
Person	29	359.357	12.392	
Time	2	43.150	21.575	
Error	56	114.425	2.043	10.56 $p = .001$
Total	87	519.185		

Mean values of intercanine widths (mms)

T1	T2	T3
33.164	34.915	34.294

Critical value using the least significant difference test ($\alpha = .05$) = 0.724

T1 - T2 = -1.751*

T1 - T3 = -1.13*

T2 - T3 = 0.621

* Exceeds the critical value

TABLE V-B

Lower Arch

Comparison of intercanine widths at the three times by analysis of variance

Source	Df.	S.S.	M.S.	F
Person	29	209.751	7.233	
Time	2	43.941	21.972	
Error	55	51.609	0.938	23.416 $p = .001$
Total	86	306.055		

Mean values of intercanine widths (mms)

T1	T2	T3
25.345	26.935	25.513

Critical value using the least significant difference test ($\alpha = .05$) = 0.490

T1 - T2 = -1.59*

T1 - T3 = -0.168

T2 - T3 = 1.422*

* Exceeds the critical value

TABLE VI-A

Upper Arch

Comparison of intermolar widths at the three times by analysis of variance

Source	Df.	S.S.	M.S.	F
Person	29	518.084	17.891	
Time	2	34.090	17.045	
Error	57	222.916	3.911	4.358 $p = .001$
Total	88	777.000		

Mean values of intermolar widths (mms)

T1	T2	T3
48.536	47.860	46.993

Critical value using the least significant difference test ($\alpha = .05$) = 1.001

T1 - T2 = 0.676

T1 - T3 = 1.54*

T2 - T3 = 0.867

* Exceeds the critical value

TABLE VI-B

Lower Arch

Comparison of intermolar widths at the three times by analysis of variance

Source	Df.	S.S.	M.S.	F.
Person	29	486.254	16.767	
Time	2	90.549	45.274	
Error	58	170.315	2.936	15.418 $p = .001$
Total	89	747.117		

Mean values of intermolar widths (mms)

T1	T2	T3
42.384	40.836	39.957

Critical value using the least significant difference test ($\alpha = .05$) = 0.867

T1 - T2 = 1.55*

T1 - T3 = 2.427*

T2 - T3 = 0.879*

* Exceeds the critical value

TABLE VII-A

Error of the Method: Mean Difference in X, Y Coordinates Between Original and Replicate Data Points (mm^2)

Upper Arch

ID	# Points (N)	$\frac{\Sigma({}^{X}i_1 - {}^{X}i_2)^2}{2N}$	$\frac{\Sigma({}^{Y}i_1 - {}^{Y}i_2)^2}{2N}$
418 - 2	16	.1139	.2475
418 - 3	16	.2450	.5772
660 - 2	16	.0768	.1710
681 - 2	16	.1060	.2458
720 - 1	18	.2954	.7500
820 - 3	16	.0380	.1261
853 - 2	16	.0753	.0285
9001 - 1	18	.0853	.0885
9006 - 2	18	.5322	1.2569
9006 - 3	18	.0449	.0603
Mean		.1613	.3552

TABLE VII-B

Error of the Method: Mean Difference in X, Y Coordinates Between Original and Replicate Data Points (mm^2)

Lower Arch

ID	# Points (N)	$\frac{\Sigma({}^{X}i_1 - {}^{X}i_2)^2}{2N}$	$\frac{\Sigma({}^{Y}i_1 - {}^{Y}i_2)^2}{2N}$
418 - 2	16	.1104	.3122
418 - 3	16	.2370	.3460
660 - 2	16	.1706	.3597
681 - 2	16	.2370	.2506
720 - 1	18	.0418	.1172
820 - 3	16	.0827	.0828
853 - 2	16	.0234	.0651
9001 - 1	18	.0451	.1618
9006 - 2	18	.2852	.6632
9006 - 3	18	.1799	.0410
ean		.1413	.2399

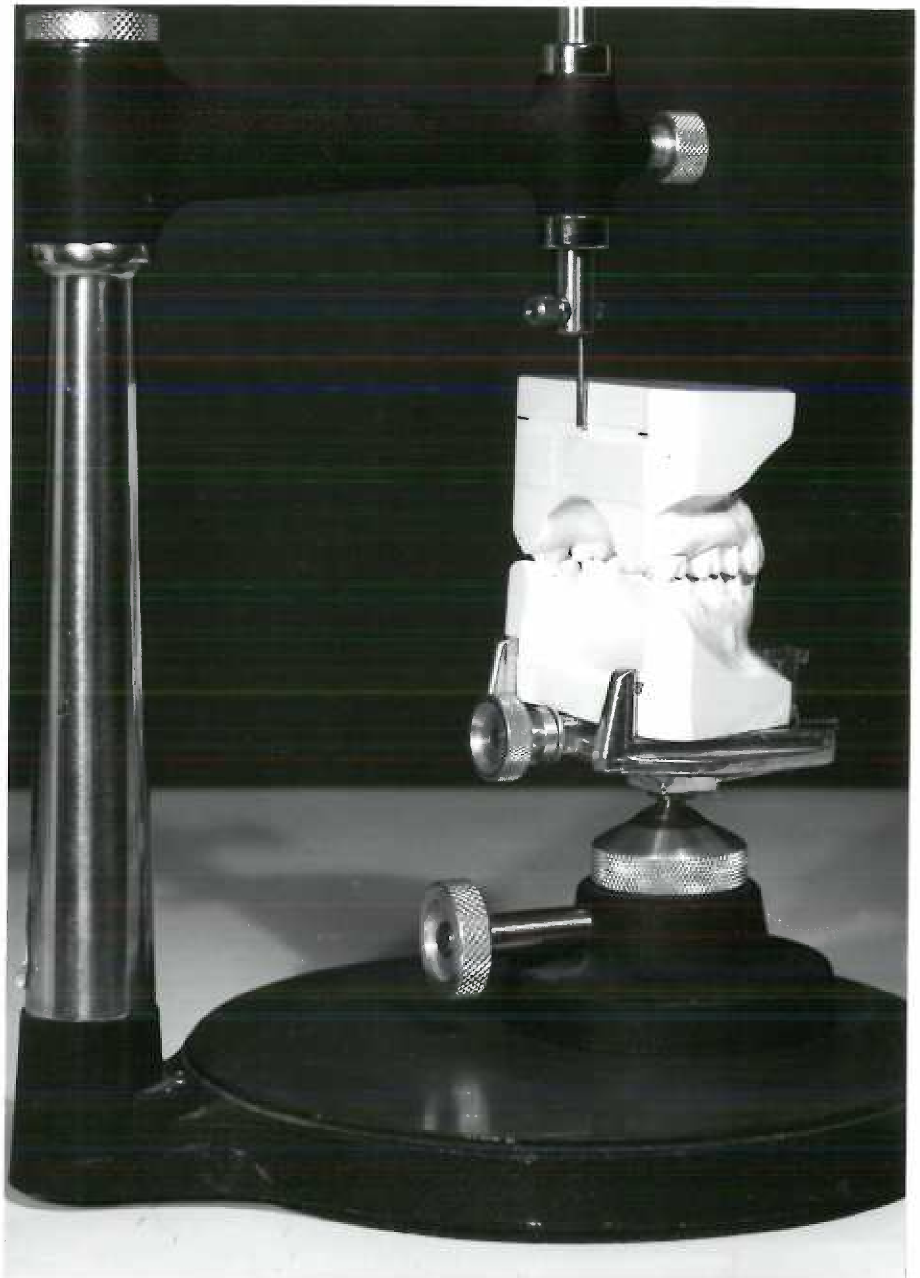


Figure 1: Lower model mounted and oriented in surveyor table. Upper model placed in centric occlusion on lower model and orientation marks drawn in on the heel.

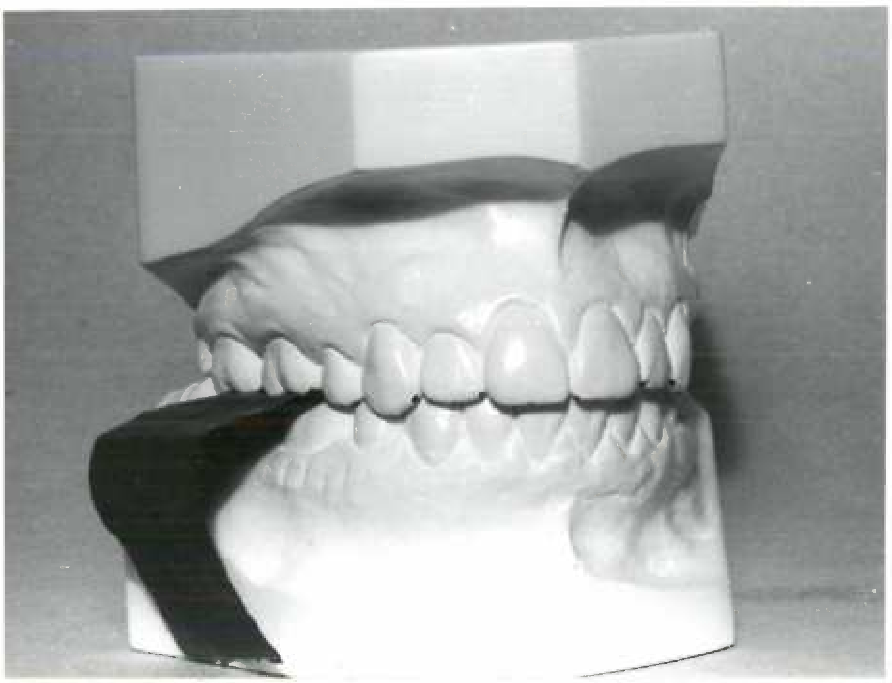


Figure 2: Orientation marks made with articulating paper placed between upper and lower models in centric occlusion.

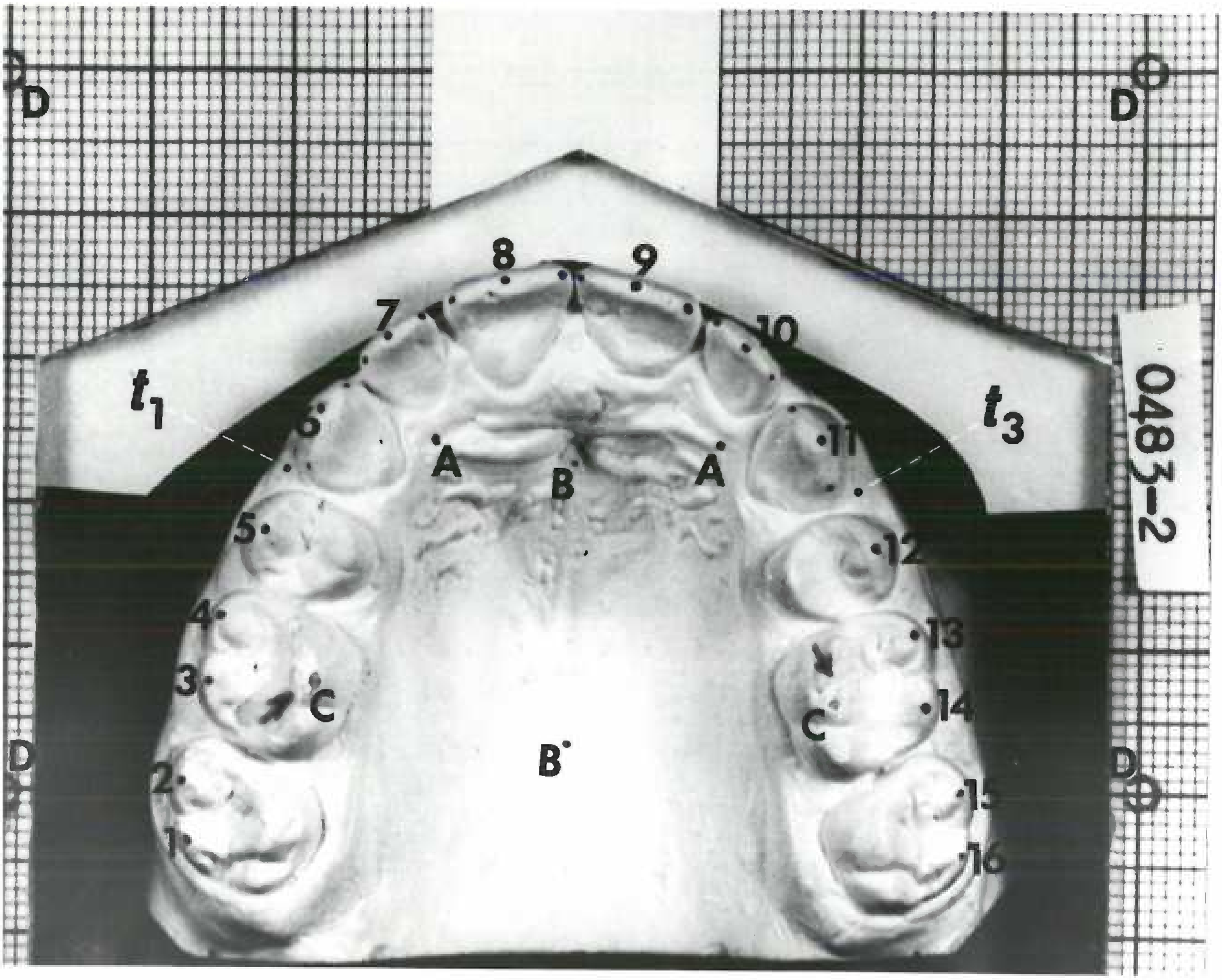


Figure 3: A representative photograph showing the marks used in orientation and digitizing.

1 - 16	Dental landmarks
A	Lateral rugae landmarks
B	Midpalatal raphe landmarks
C	Articulation marks used to establish reference line
D	Fiducial points
t_1	Spline knot #1
t_3	Spline knot #3

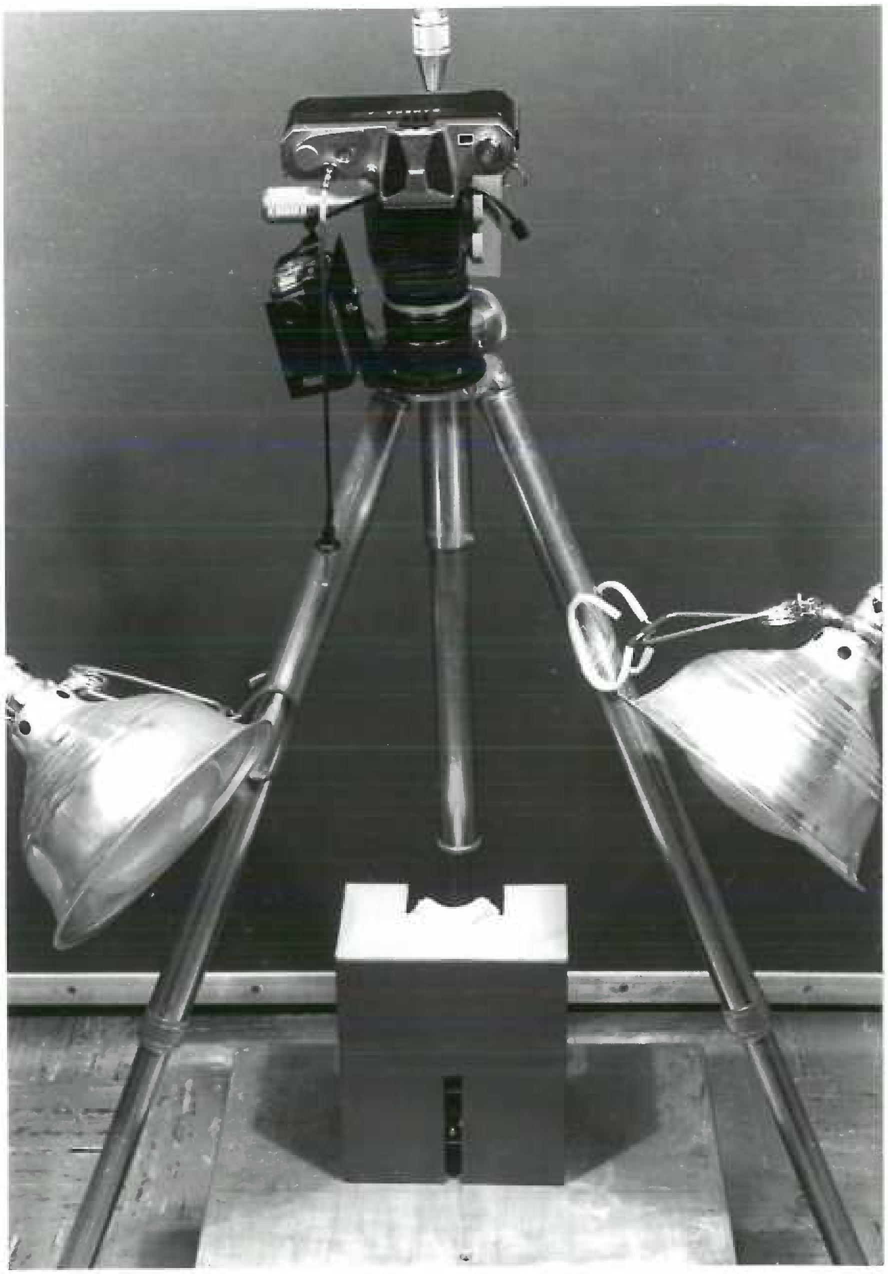
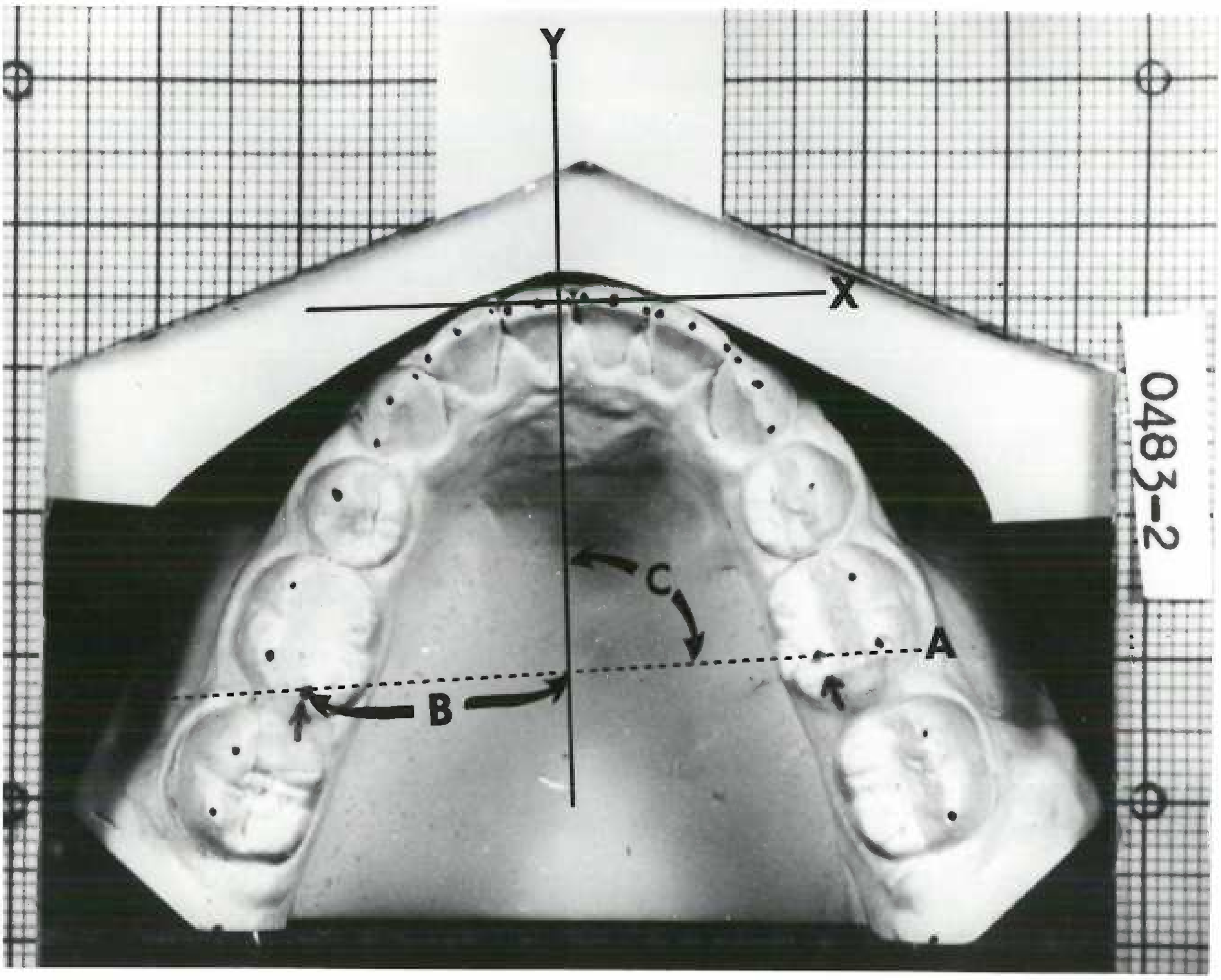


Figure 4: Nikkormat 35 mm camera with bellows and 100 mm short mount lens mounted on tripod. The model to be photographed is placed on a laboratory jack and raised so that the occlusal plane is level with the top of the orientation table.



A - Reference line passing through articulation marks

B - Distance from articulation mark to intersection of Y axis with reference line

C - Angle between reference line and Y axis

X - X axis passing through most anterior incisor midpoint

Y - Y axis passing through midpalatal raphe landmarks in the upper arch and transferred to the lower arch.

No.	Source	Sex	Classification	Age* at Tx Start	Age at Tx End	Post Ret. Age	Post Tx Period	Post Ret. Period
0403	U. O.	F	I	12 - 6	14 - 8	25 - 4	10 - 8	7 - 2
0409	U. O.	F	II - 1	9 - 3	13 - 9	22 - 0	10 - 5	8 - 6
0483	U. O.	F	II - 2	14 - 4	16 - 6	25 - 9	9 - 3	8 - 9
0412	U. O.	F	II - 1	9 - 11	16 - 11	22 - 8	6 - 0	4 - 9
0720	U. O.	F	I	17 - 2	19 - 9	26 - 8	6 - 11	4 - 6
0428	U. O.	F	II - 1	9 - 10	13 - 6	22 - 7	9 - 1	4 - 6
0660	U. O.	F	II - 1 (sub)	11 - 0	13 - 8	21 - 4	7 - 8	7 - 8
0681	U. O.	F	II - 1	13 - 9	15 - 10	23 - 11	8 - 1	8 - 1
0776	U. O.	F	II - 2	12 - 9	16 - 1	22 - 0	5 - 11	5 - 11
0853	U. O.	M	II - 1	11 - 4	14 - 2	20 - 0	4 - 10	4 - 10
0818	U. O.	M	I	11 - 5	14 - 1	20 - 5	6 - 4	4 - 3
0820	U. O.	M	II - 1	12 - 1	16 - 2	20 - 5	4 - 3	4 - 3
0540	U. O.	F	I	12 - 7	15 - 4	27 - 8	12 - 4	10 - 8
0554	U. O.	F	II - 1 (sub)	13 - 6	16 - 5	28 - 6	12 - 1	11 - 2
0406	U. O.	F	I	12 - 10	15 - 4	29 - 6	14 - 2	11 - 3
0418	U. O.	F	I	12 - 7	16 - 0	29 - 10	13 - 10	10 - 9
0416	U. O.	F	I	12 - 0	16 - 0	28 - 7	12 - 7	11 - 9

* (yr - mo)

No.	Source	Sex	Classification	Age at Tx Start	Age at Tx End	Post Ret. Age	Post Tx Period	Post Ret. Period
0503	U. O.	M	II - 2	13 - 7	15 - 9	28 - 10	13 - 1	10 - 10
0534	U. O.	M	II - 2	12 - 6	16 - 7	27 - 8	11 - 1	9 - 1
0600	U. O.	F	I	13 - 0	14 - 10	27 - 6	13 - 6	11 - 6
0639	U. O.	M	I	12 - 8	15 - 4	26 - 11	11 - 7	10 - 1
9001	U. W.	F	II - 1	12 - 0	13 - 8	29 - 5	15 - 9	11 - 0
9002	U. W.	F	II - 1	12 - 11	15 - 5	30 - 7	15 - 2	14 - 4
9003	U. W.	F	I	11 - 8	14 - 4	26 - 7	12 - 3	11 - 3
9004	U. W.	M	I	11 - 5	13 - 5	28 - 11	15 - 6	13 - 4
9005	U. W.	F	III (Pseudo)	12 - 3	13 - 3	26 - 5	13 - 2	12 - 5
9006	U. W.	M	II - 1	8 - 11	10 - 5	20 - 7	10 - 2	5 - 4
9007	U. W.	F	II - 1	16 - 10	18 - 7	30 - 2	13 - 4	12 - 9
9008	U. W.	M	II - 1	13 - 0	15 - 2	29 - 0	13 - 10	13 - 4
9009	U. W.	M	II - 1	12 - 10	15 - 11	27 - 3	11 - 4	11 - 2

U. O. - University of Oregon School of Dentistry
U. W. - University of Washington School of Dentistry