

STATISTICAL MEASUREMENTS OF SPECKLE PROPAGATION
THROUGH THE TURBULENT ATMOSPHERE

by

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ABSTRACT

This study was concerned with the effects of atmospheric turbulence on propagating speckle fields. Measurements of normalized variance and covariance were made at two path lengths under various levels of turbulence strength.

The results are compared to recently published theoretical formulations. The theory, developed by Lee, Holmes, and Kerr, contains general formulations for normalized variance and covariance of intensity for single frequency, single mode laser illumination of a diffuse target through turbulence. By assuming that phase perturbation is the dominant effect of turbulence and that the fields at the receiver are jointly gaussian, simple closed form expressions are obtained.

The covariance measurements described herein indicate that the additive amplitude perturbation terms in the general formulation may be very important in moderate to high turbulence conditions. It appears that the simple gaussian assumption theory predicts accurate covariance scales only in low turbulence conditions (where $\rho_0 > \sqrt{L/K}$).

Due to inadequate temporal coherence of the illuminating laser, the measured normalized variance was lower than expected. As a result it was not possible to make detailed quantitative comparisons to either the general or simplified theory. However, the measurements do show a qualitative agreement with the general theory.

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I. INTRODUCTION

Recent investigations involving Remote Wind Sensing^{1,2,3} and Coherent Adaptive Transmitters have led to a need to understand the effects of the atmosphere on propagating speckle fields.

Previous measurements of speckle statistics have been primarily concerned with the nature of the target surface, propagation of speckle without turbulence and the effects of speckle on image quality. Speckle propagation through the atmosphere on a vertical path has been studied in the context of speckle interferometry.⁴

The purpose of the experiments described herein was to measure the statistical properties of laser light propagating to and from a diffuse target on a horizontal path through a turbulent atmosphere.

The particular aim of this effort was to test experimentally some of the recent theoretical results obtained by Lee, Holmes, and Kerr.^{3,5} They found simple closed form expressions for the normalized covariance function of intensity for focused and collimated beams utilizing two basic assumptions.

1. Phase perturbation is the dominant effect of turbulence.
2. The fields at the receiver (which can be shown to be marginally gaussian) are assumed to be jointly gaussian.

These same assumptions lead to the result that the normalized variance of intensity is unity. The first assumption leads to the prediction that the probability density function of the intensity is exponential.

An attempt was made to determine the range of validity of the above assumptions by measuring the normalized variance and covariance in various realms of turbulence and path length.

II. EXPERIMENTAL DESIGN

The general experimental layout is shown in Figure 1. The transmitter consists of a single mode laser and optics to form an expanded beam and to allow for focusing at various distances. The transmitted beam is modulated to isolate the signal from background light.

The receiver consists of two separate detectors mounted on an optical bed which allows measurements to be made at various detector separations. The signals are demodulated and recorded for computer analysis.

The target consists of scotchlite which acts as a random phase reflector to produce a speckle pattern yet is directional and provides enough energy for an adequate signal to noise ratio. The path of propagation is at a height of 2 meters over a flat cultivated field. This path provides uniform turbulence that can be readily characterized.

A. Transmitter

The laser used in these experiments is a Coherent Radiation Model 52 which provides about one watt of power at the 488 nm line. The light from the laser is amplitude modulated at 100 kHz by passing through a Morvue Pem-3 photo-elastic modulator and a calcite polarizer. The off axis beam from the polarizer is blocked and the scattered light is detected by a silicon photodiode. This provides a 100 kHz local oscillator for use in the synchronous demodulators. The on axis beam is passed through a convex focusing lens, a spatial filter, and a three inch diameter objective lens. The resultant beam is steered to the target by a large flat mirror (see Figure 2).



Figure 1 - Target-Illuminator Configuration

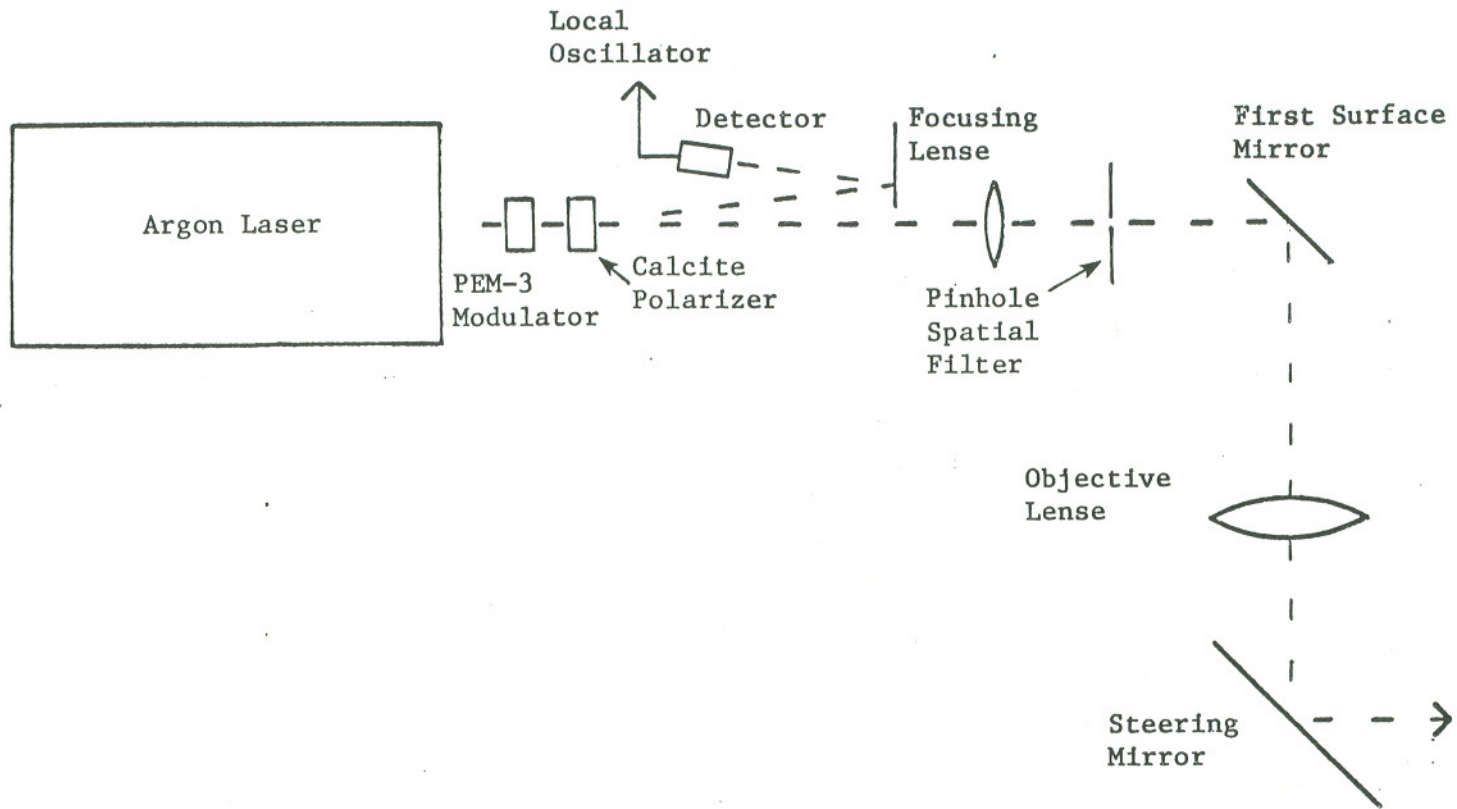


Figure 2 - Transmitter Schematic

At the output window the measured $1/e$ point of the intensity profile is 1.35 cm when focused at 910 meters and 1.84 cm when focused at 500 meters. The modulation depth is about 80% and the average transmitted power is 110 milliwatts.

B. Receiver

Each of the two detectors consists of an S-20 photomultiplier tube isolated from excess background light by a 10 nm optical bandpass filter and field of view limiting apertures as shown in Figure 3. As shown in Figure 4, the current from the PM tube is fed to the inverting input of an op-amp and the resulting signal is passed through a bandpass filter to the synchronous demodulator. The output from the demodulator is lowpass filtered and recorded on an HP-3960 instrumentation tape recorder. An estimate of the time averaged covariance is obtained using a covariance meter.

C. Target

Scotchlite (3M Sprint Marking Paper) was chosen as the target material. It provides a directional return which realizes a gain in signal strength of about one thousand to one in comparison with a perfect Lambertian reflector. This material also imparts a random phase on the reflected light and therefore produces a speckle pattern when illuminated with monochromatic light, characteristic of a diffuse surface.

Experiments were performed in the laboratory at 632.8 nm to determine the suitability of the target material. When the illuminating spot was large enough to cover a reasonable number of the scotchlite

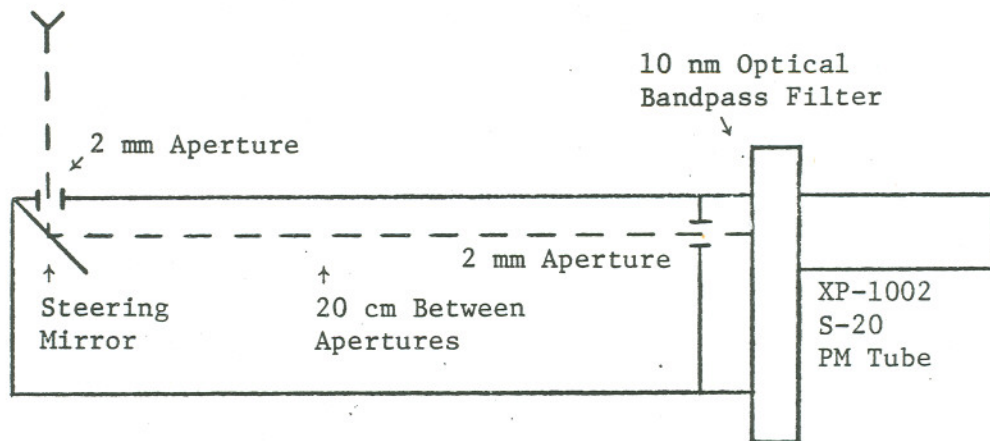


Figure 3 - Detector Schematic

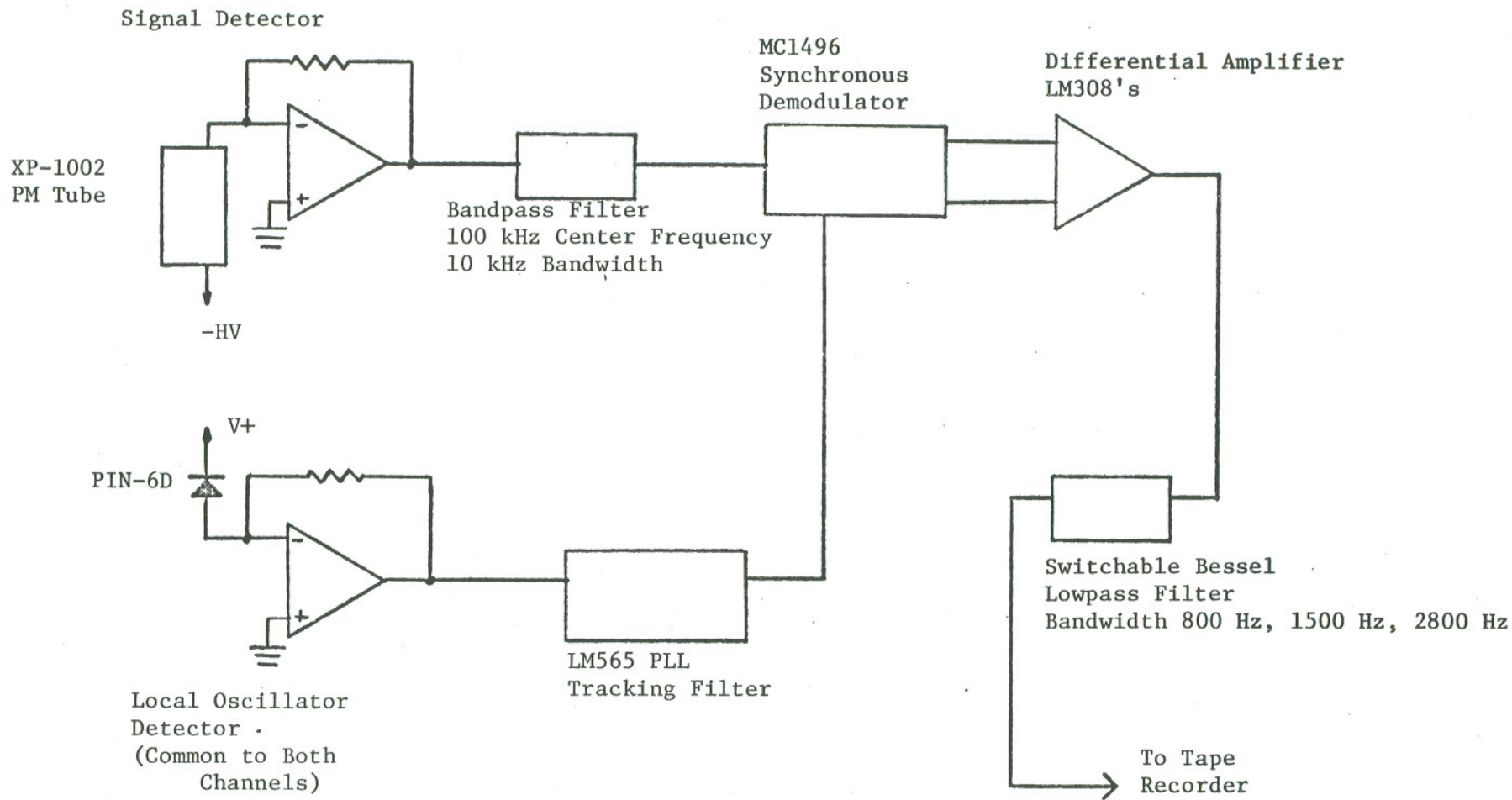


Figure 4 - Functional Schematic of One Receiver Channel

'beads' a random speckle pattern resulted with normalized variance of intensity equal to unity.

In order to obtain good measurements of the statistics of the intensity, it is necessary to observe many 'speckles'. Under high turbulence conditions this is no problem because the intensity seen by each detector is constantly evolving as the atmosphere changes. Under low turbulence conditions a speckle pattern resulting from illuminating a stationary target is nearly stationary, and two fixed detectors cannot measure the statistics of this pattern. This problem was overcome by using a large rotating cylinder for a target so that the speckle pattern moves continuously past the detectors. Care was taken that the target moved slowly enough so the signal spectrum remained entirely within the demodulated bandwidth of the receiver.

D. Field Site Instrumentation

Turbulence strength was measured by two well established methods. A Contel, Inc. MT-2 Microthermal Amplifier was used to measure small scale temperature fluctuations from which C_n^2 can be obtained.⁶ At the same time, a Cambell Scientific CA-9 Space Averaging Anemometer was used to measure the log amplitude standard deviation (σ_x) from which C_n^2 can also be obtained.⁷ With proper correction of σ_x for aperture averaging effects, the resultant values of C_n^2 from the two methods were found to agree consistently.⁸ The CA-9 also provided a measure of the transverse wind velocity.

E. Data Collection and Analysis

Each data set consisted of a series of 90 second recordings of

both channels at various detector separations. For each set, observations of atmospheric conditions including temperature measurements were recorded on a data sheet. During each 90 second recording a reading of turbulence was taken from both the MT-2 and CA-9. The covariance meter was also read at each separation.

With each data set, a background reading was recorded. This reading was obtained by turning off the modulator and attenuating the laser beam to obtain the same average transmitted power. While an external local oscillator was provided for the demodulators, the noise in each channel was recorded. Using this method an accurate measure of the signal to noise ratio (SNR) was obtained and the DC offset in each channel was determined.

The basic data processing system is shown in Figure 5. This system has the capability of sampling both channels at a 2000 Hz rate. There is software for digitally approximating the normalized covariance between the two channels, for printing out the probability density function and the first four moments for one channel at a time, and for doing spectral analysis by means of a fast fourier transform algorithm. This system was developed over the past few years and has been used extensively by previous investigators.⁶

The normalized covariance function is calculated as follows.

$$C_{IN} = \frac{\langle (z_1 - \bar{z}_1) (z_2 - \bar{z}_2) \rangle}{\sigma_{z1} \sigma_{z2}}$$

In this equation z_1 and z_2 are the sampled voltages on each channel and σ_{z1} , σ_{z2} are their respective standard deviations.

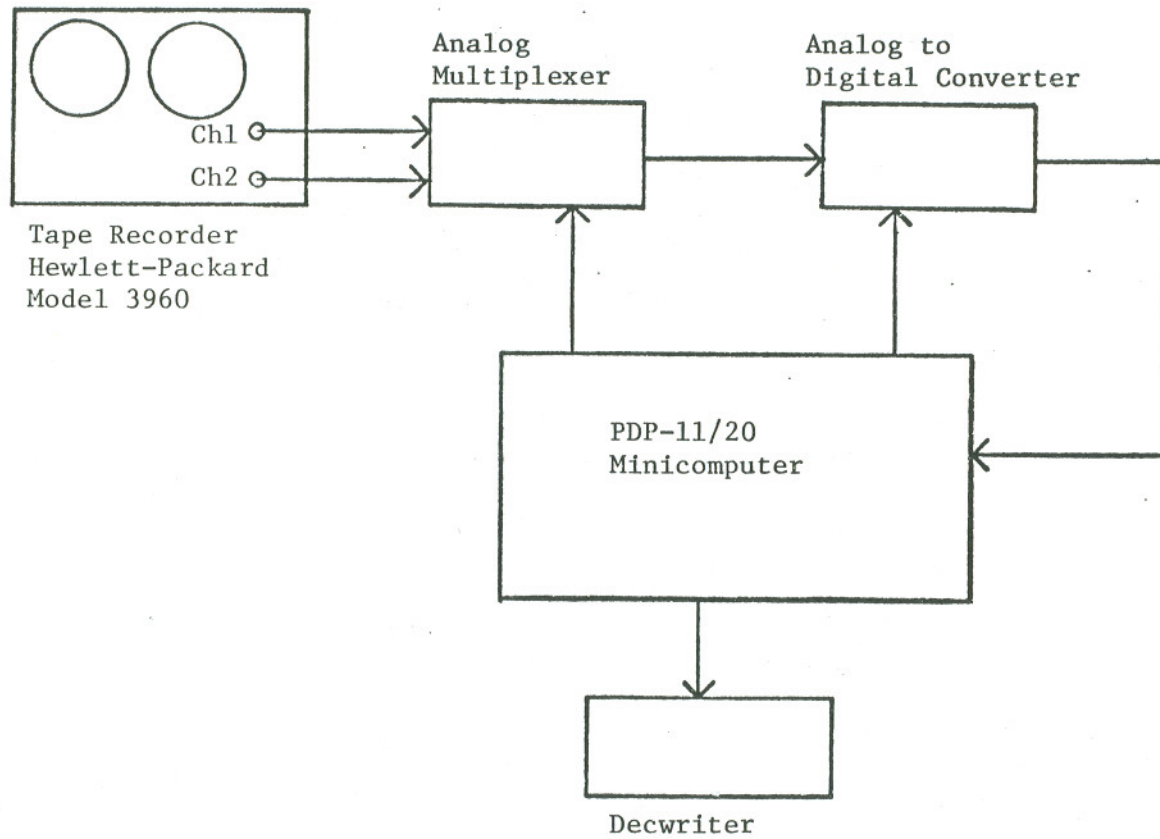


Figure 5 - Data Processing System

The normalized variance is obtained from the first two moments of the probability density function and the mean of the background signal.

$$\sigma_{IN}^2 = \frac{\langle z^2 \rangle - \langle z \rangle^2}{(\langle z \rangle - \langle y \rangle)^2}$$

Here z represents the sampled signal and y represents the noise and DC offset of the electronics.

F. Signal to Noise Ratio and Measurement Accuracy

When light is scattered from a Lambertian reflector, the average received power is given by

$$P_R = \frac{P_T T_R T_F}{4} \cdot (d/L)^2$$

where,

P_T = average transmitted power

T_R = target reflectance

T_F = passband transmittance of optical filter

d = receiver aperture diameter

L = distance to target.

If $T_R = 1000$ (the relative gain of the scotchlite) is used, a good estimate of received power in the experimental setup is obtained.

Assuming a power incident on the target from the sun of 300 watts/meter²-micron, the received background power is given by

$$P_B = \frac{300 T_R T_F (d^2/s)^2 BW_0}{16}$$

where,

s = distance between the two apertures

BW_0 = optical bandwidth in microns

Note that since the sunlight strikes the scotchlite at an angle off the optical axis, T_R will be a constant less than unity for the

purpose of calculating background power.

The signal to noise ratio for shot noise limited operation is given by

$$\text{SNR} = \frac{i_s^2}{i_n^2} = \frac{P_s^2 \rho}{2eB(P_R + P_B)}$$

where, ρ = responsivity of the photocathode
 e = electronic charge
 B = demodulated bandwidth
 P_S = power contained in the 100 kHz modulation (with 80% modulation depth $P_S = .47 P_R$).

Using the following values,

$P_T = .11$ watts
 $d = 2 \times 10^{-3}$ meters
 $T_F = 0.5$
 $L = 910$ meters
 $s = 0.2$ meters
 $BW_0 = 0.01$ microns
 $\rho = 0.07$ amps/watt
 $B = 3 \times 10^3$ hertz
 $e = 1.6 \times 10^{-19}$ coulombs

these results are obtained.

$P_R = 6.6 \times 10^{-11}$ watts
 $P_B = 3.75 \times 10^{-11}$ watts
 $\text{SNR} = 678$

Note that the dark current measured at the anode is 5 nA. The total anode current $I_a = \rho M(P_B + P_R) = 6.7$ microamperes, therefore the noise contributed by the PM tube is negligible ($M =$ current multiplication in the tube).

Measured signal to noise ratio from a typical data set is 130. This additional reduction in SNR can be accounted for by the noise added in the process of recording and reproducing the data.

In the following section the effect of SNR on the accuracy of measuring normalized variance and covariance is analyzed.

Let, $x =$ signal

$y =$ noise (including DC offset)

$z = x + y$

the measured normalized variance is

$$\sigma_{INM}^2 = \frac{\langle z^2 \rangle - \langle z \rangle^2}{(\langle z \rangle - \langle y \rangle)^2} = \frac{\sigma_x^2 + \sigma_y^2 + 2r\sigma_x\sigma_y}{\langle x \rangle^2}$$

where r is the correlation coefficient between the signal and the noise.

$$\sigma_{INM}^2 = \sigma_{IN}^2 + \frac{\sigma_y^2}{\langle x \rangle^2} + \frac{2r\sigma_x\sigma_y}{\langle x \rangle^2}$$

$$\sigma_{INM}^2 = \sigma_{IN}^2 \left(1 + \frac{\sigma_y^2}{\sigma_x^2} + \frac{2r\sigma_y}{\sigma_x} \right)$$

$$SNR = \frac{\langle x \rangle^2}{\sigma_y^2} \approx \frac{\sigma_x^2}{\sigma_y^2}$$

since

$$\sigma_{IN}^2 = \frac{\sigma_x^2}{\langle x \rangle^2} \approx 1$$

$$\sigma_{\text{INM}}^2 \approx \sigma_{\text{IN}}^2 \left(1 + \frac{1}{\text{SNR}} + \frac{2r}{\sqrt{\text{SNR}}} \right) = \sigma_{\text{IN}}^2 (1+\epsilon)$$

The worst case condition occurs when $r=1$. Using the measured SNR of 130, $\epsilon = .18$. Since the only noise likely to be highly correlated with the signal is the signal shot noise, a more reasonable yet still very conservative value for r is 0.1. This estimate results in $\epsilon = .025$. Therefore the measurement accuracy should be better than 3% as long as adequate SNR is maintained.

In order to calculate the effect of SNR on the accuracy of normalized covariance measurements, assume the two channels have the same gain and SNR such that,

$$\sigma_{x1} = \sigma_{x2} = \sigma_x, \quad \langle x_1 \rangle = \langle x_2 \rangle = \bar{x}$$

$$C_{\text{IN}} = \frac{\langle (x_1 - \bar{x})(x_2 - \bar{x}) \rangle}{\sigma_x^2}$$

Let $z_1 = x_1 + y_1$, $z_2 = x_2 + y_2$; $y_1, y_2 =$ zero mean random variables; $\sigma_{y1}, \sigma_{y2} = \sigma_y$. The measured normalized covariance in the presence of noise is given by the following expression.

$$C_{\text{INM}} = \frac{\langle (z_1 - \bar{z})(z_2 - \bar{z}) \rangle}{\sigma_z^2} = \frac{\langle x_1 x_2 \rangle - \bar{x}^2 + \langle x_1 y_2 \rangle + \langle x_2 y_1 \rangle + \langle y_1 y_2 \rangle}{\sigma_x^2 + \sigma_y^2 + 2r\sigma_x\sigma_y}$$

If it is assumed that the noise in one channel is not correlated with the signal or noise of the other channel, the following result is obtained.

$$C_{INM} = \frac{C_{IN}}{1 + \frac{\frac{y}{\sigma_x^2}}{\sigma_x^2} + 2r \frac{y}{\sigma_x}} = \frac{C_{IN}}{1 + \frac{1}{SNR} + \frac{2r}{\sqrt{SNR}}}$$

$$C_{IN} = C_{INM} (1 + \epsilon)$$

The error terms and the estimated error for the covariance measurement are equal and opposite to those of the variance. While noise tends to increase the measured variance, it tends to decrease the measured normalized covariance.

The most critical parameter necessary for obtaining an accurate value for the normalized variance is the corrected mean used for normalization. The effect of any uncorrected DC offset on the measured normalized variance is shown below.

Let x = signal, and a = DC offset. The variance measured in the presence of the offset is given below.

$$\sigma_{INM}^2 = \frac{\langle (x+a)^2 \rangle - \langle x+a \rangle^2}{\langle x+a \rangle^2}$$

$$\sigma_{INM}^2 = \frac{\langle x^2 \rangle - \bar{x}^2}{\bar{x}^2 \left(1 + \frac{2a}{x} + \frac{a^2}{x^2} \right)} = \frac{\sigma_{IN}^2}{1 + 2 \frac{a}{x} + \left(\frac{a}{x} \right)^2}$$

$$\sigma_{IN}^2 = \sigma_{INM}^2 \left(1 + 2 \frac{a}{x} + \left(\frac{a}{x} \right)^2 \right)$$

It is apparent that substantial errors in the measurement can result unless the uncorrected offset is a very small fraction of the mean signal. This error could result from inaccurate measurement of the DC offset, detection of spurious modulated light scattered from the optics, or a specular return from the target in addition to the

desired diffuse return. The first two sources were readily eliminated by careful experimental procedure. The latter source required careful characterization and control of the target.

The measured normalized covariance in the presence of an offset in one channel is shown below.

$$C'_{IN} = \frac{\langle (x_1 + a - \langle x_1 + a \rangle) (x_2 - \langle x_2 \rangle) \rangle}{\sigma_{x_1+a} \sigma_{x_2}}$$

$$\sigma_{x_1+a} = \sigma_{x_1}$$

$$C'_{IN} = \frac{\langle (x_1 - \bar{x}_1) (x_2 - \bar{x}_2) \rangle}{\sigma_{x_1} \sigma_{x_2}} = C_{IN}$$

An additive DC term has no effect on the covariance measurement.

III. EXPERIMENTAL RESULTS

A. Theoretical Discussion

A theoretical treatment of the propagation of 'speckle' through atmospheric turbulence is contained in References 3 and 5. The derivations are based on a geometry as shown previously in Figure 1, where a TEM₀₀ source illuminates a perfectly diffuse target and the scattered radiation is detected in the transmitter plane.

General formulations for the variance and covariance were developed and the phase dominance and jointly gaussian fields assumptions were utilized to obtain the following closed form solutions for the focused and collimated configurations. Variables in these equations are identified in Table I.

Normalized Covariance of Intensity

$$C_{IN}(p) = \frac{C_I(p)}{\sigma_I^2} = \exp \left\{ -\frac{p^2}{2\alpha_o^2} - 4\left(\frac{p}{\rho_o}\right)^{5/3} \right\}$$

(focused case)

$$C_{IN}(\rho) = \exp \left\{ -4\left(\frac{p}{\rho_o}\right)^{5/3} - \frac{1}{2} \left[\frac{1}{\alpha_o^2} + \left(\frac{k\alpha_o}{K}\right)^2 \right] p^2 \right\}$$

(collimated case)

Normalized Variance of Intensity

$$\sigma_{IN}^2 = \frac{\sigma_I^2}{\langle I \rangle^2} = 1$$

(arbitrary focus)

In addition Lee⁹ has derived an equation for normalized covariance in arbitrary focus.

TABLE I - Definition of Variables

p	=	Separation between detectors
α_0	=	Radius of transmitted beam (1/e point of intensity)
k	=	Optical wave number
L	=	Distance from receiver-transmitter plane to target
F	=	Focal length of transmitted beam
ρ_0	=	Turbulence induced covariance length = $(0.545625 C_n^2 L k^2)^{-3/5}$
C_n^2	=	Structure constant of turbulence

$$C_{IN}(\rho) = \exp \left\{ -4 \left(\frac{p}{\rho_0} \right)^{5/3} - \frac{p^2}{2\alpha_0^2} - \frac{1}{2} \left(1 - \frac{L}{F} \right)^2 \left(\frac{k\alpha_0 p}{L} \right)^2 \right\}$$

The equations listed above are for point detectors. The theoretical covariance curves used for comparison with experiment were calculated using a computer program performing a numerical integration to compensate for aperture averaging.^{9,10}

B. Covariance Results

The experimental curves of normalized covariance vs. detector separation are shown in Figures 6 through 12. Figures 6 through 9 show a transition from low to high turbulence at a distance of 910 meters, with the transmitter focused on the target. The two lower turbulence curves agree with the theoretical curves very well, deviating only at the large separations. Figure 6 illustrates the realm where the covariance length is determined by the 'target speckles' (covariance length $\approx 2\alpha_0$). In Figure 7 the covariance length has decreased under the influence of the atmospheric phase perturbation term

$$\left(\exp \left\{ -4 \left(\frac{p}{\rho_0} \right)^{5/3} \right\} = e^{-2} \text{ when } p = .66 \rho_0 \right).$$

The high turbulence cases show very little change in the width of the covariance curves while the theory predicts a rapid decrease in covariance length with increasing turbulence.

Figures 10 and 11 represent data taken at 500 meters in the focused configuration. Again it is apparent that while the theory predicts a rapid decrease in covariance length with increasing turbulence, the experiment shows very little change.

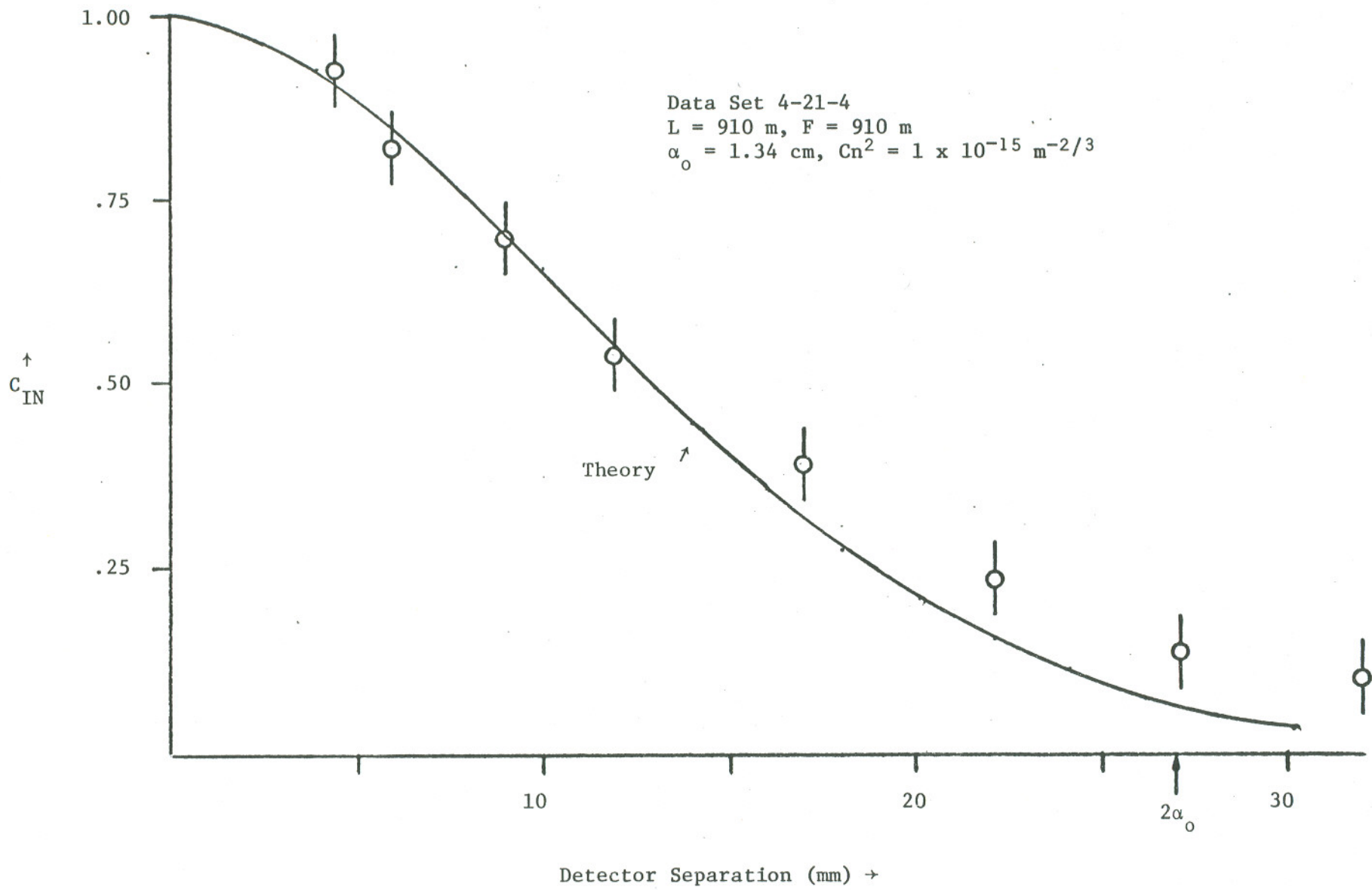


Figure 6 - Normalized Covariance Data

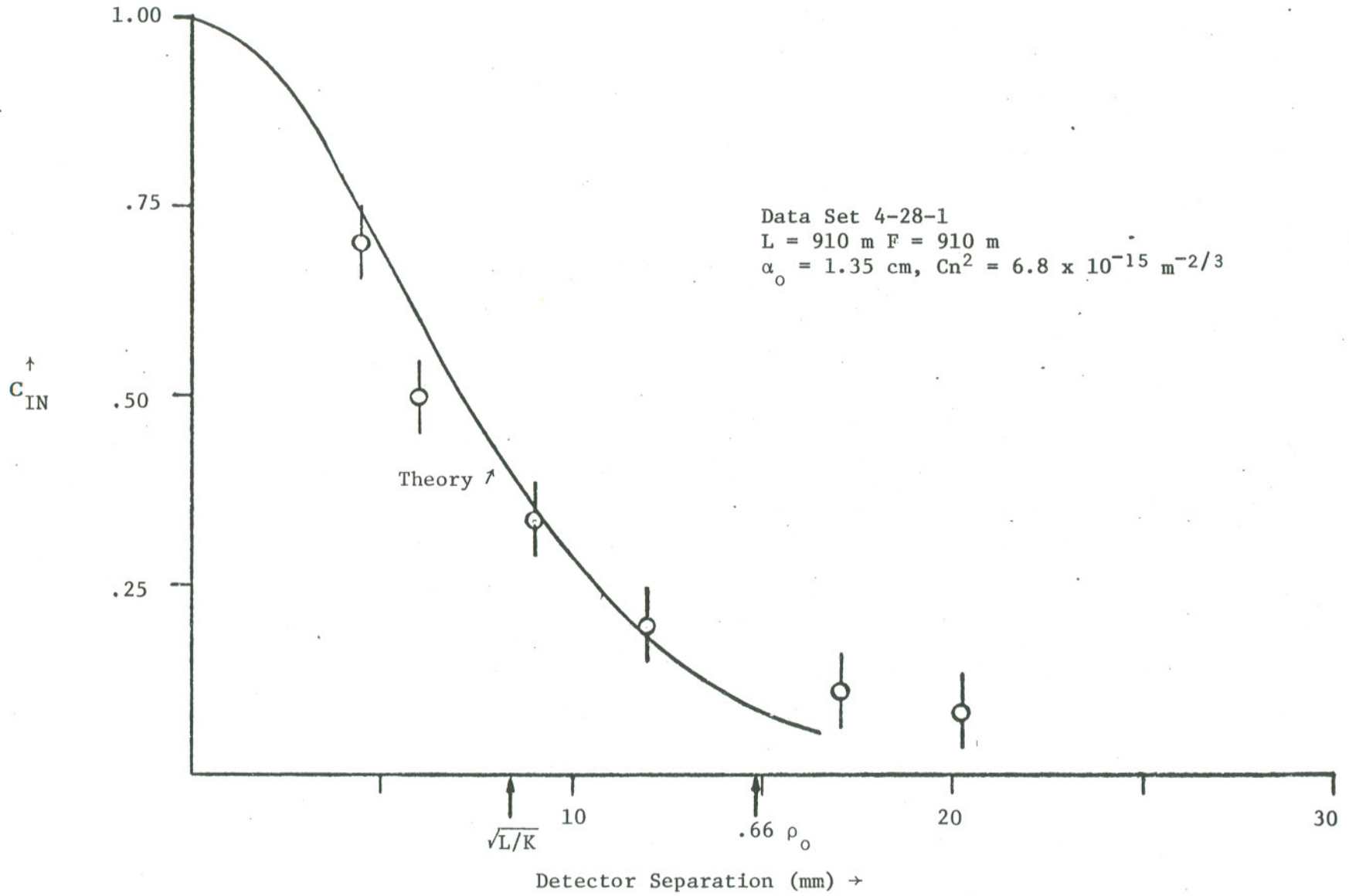


Figure 7 - Normalized Covariance Data

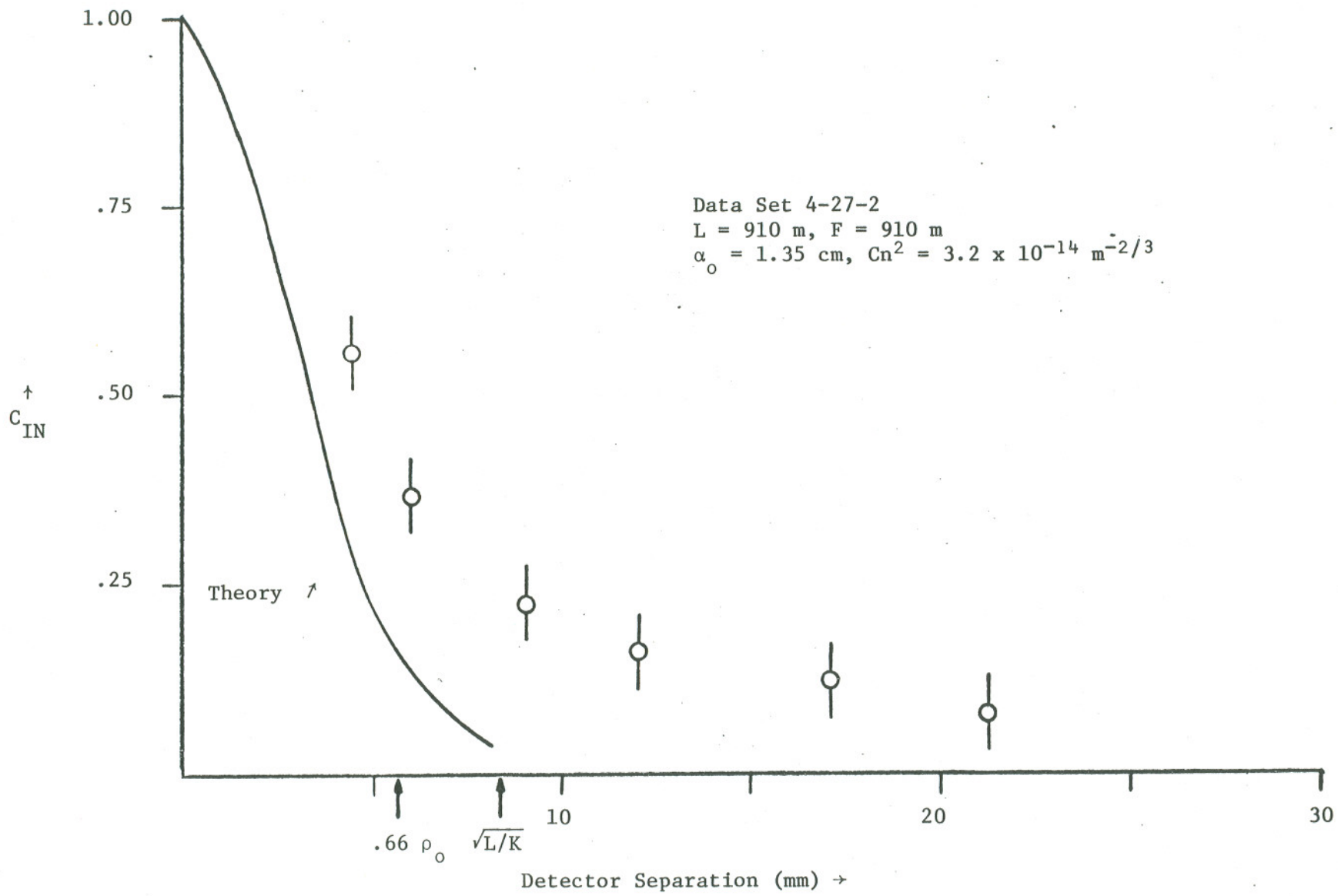


Figure 8 - Normalized Covariance Data

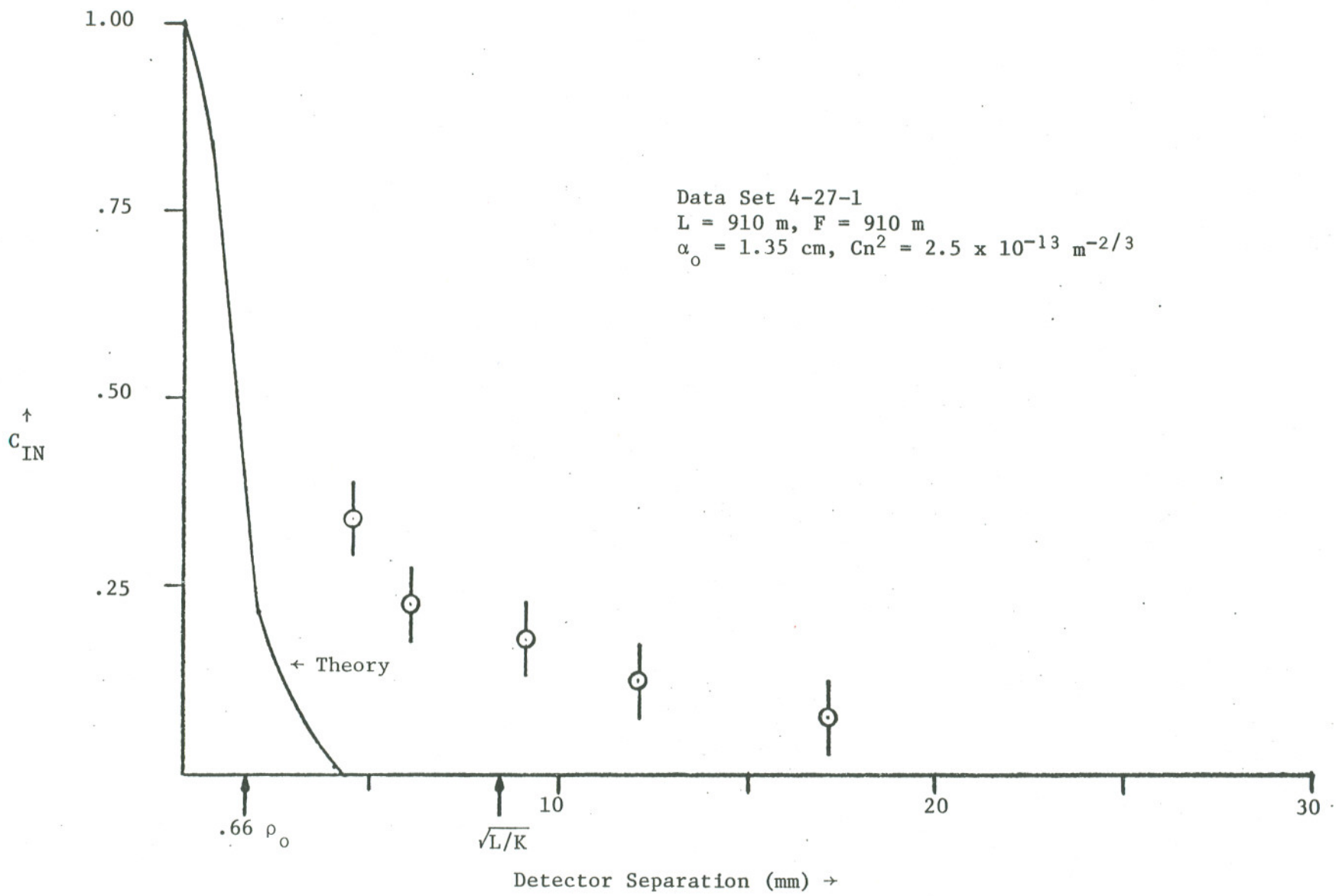


Figure 9 - Normalized Covariance Data

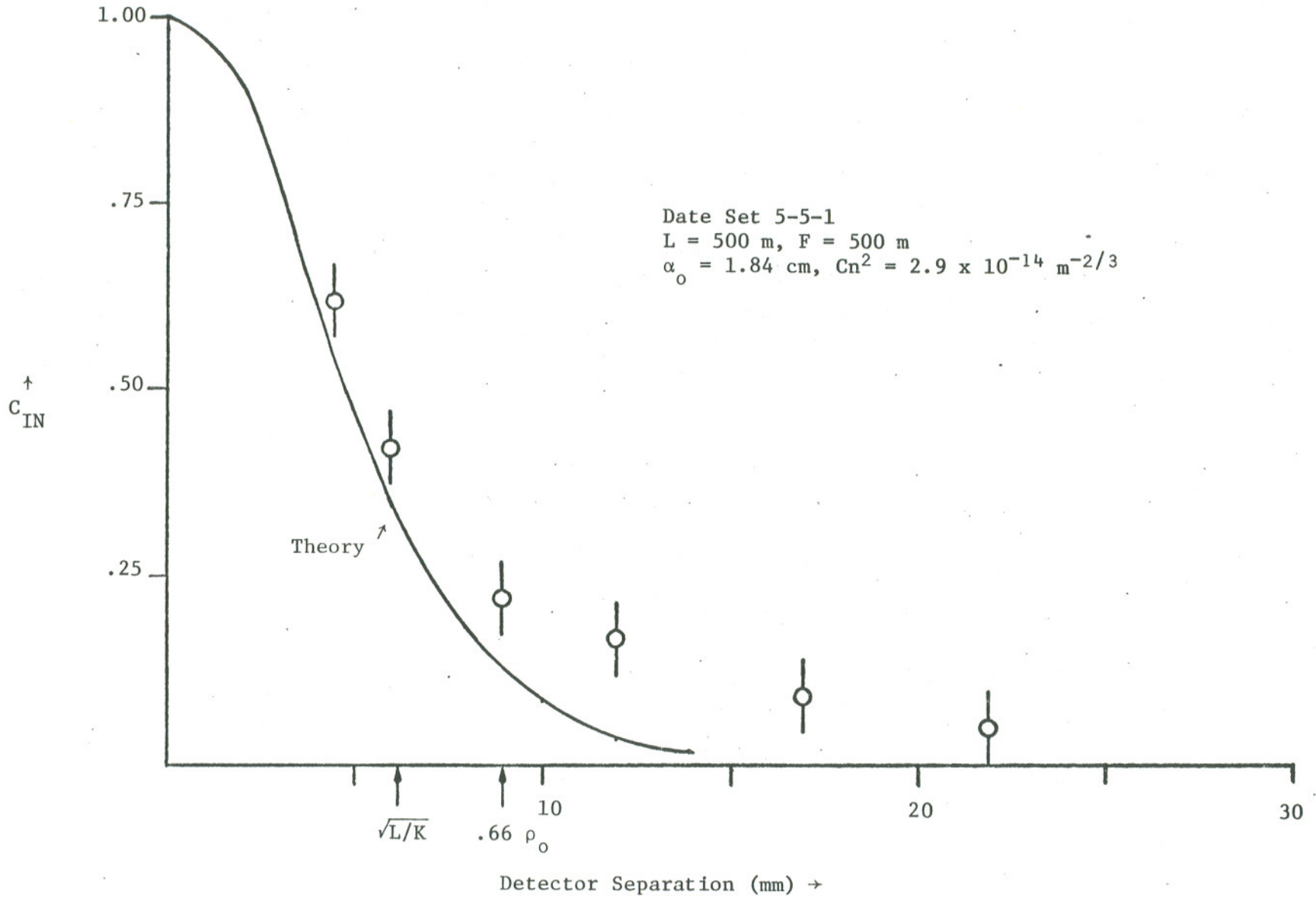


Figure 10 - Normalized Covariance Data

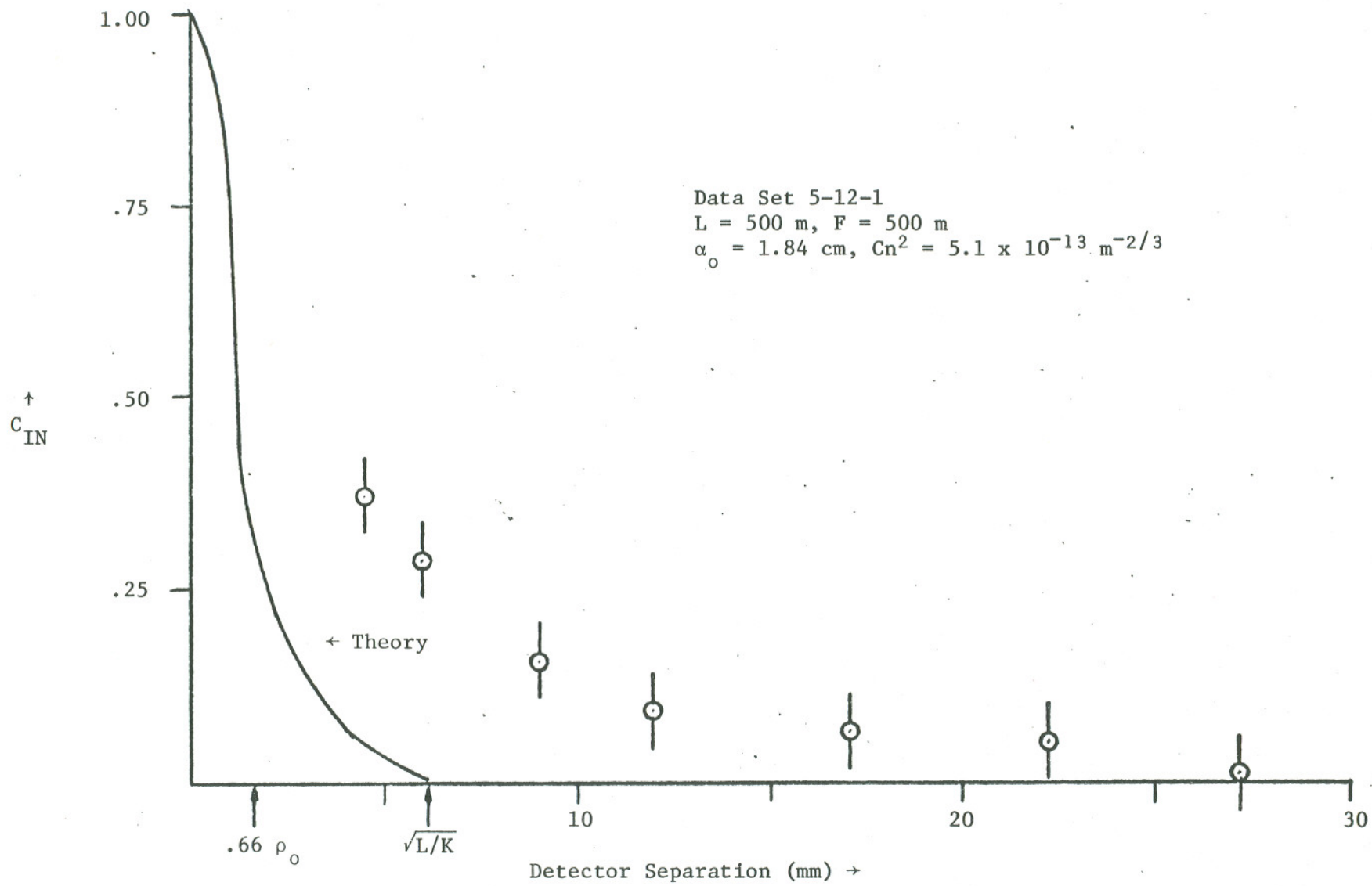


Figure 11 - Normalized Covariance Data

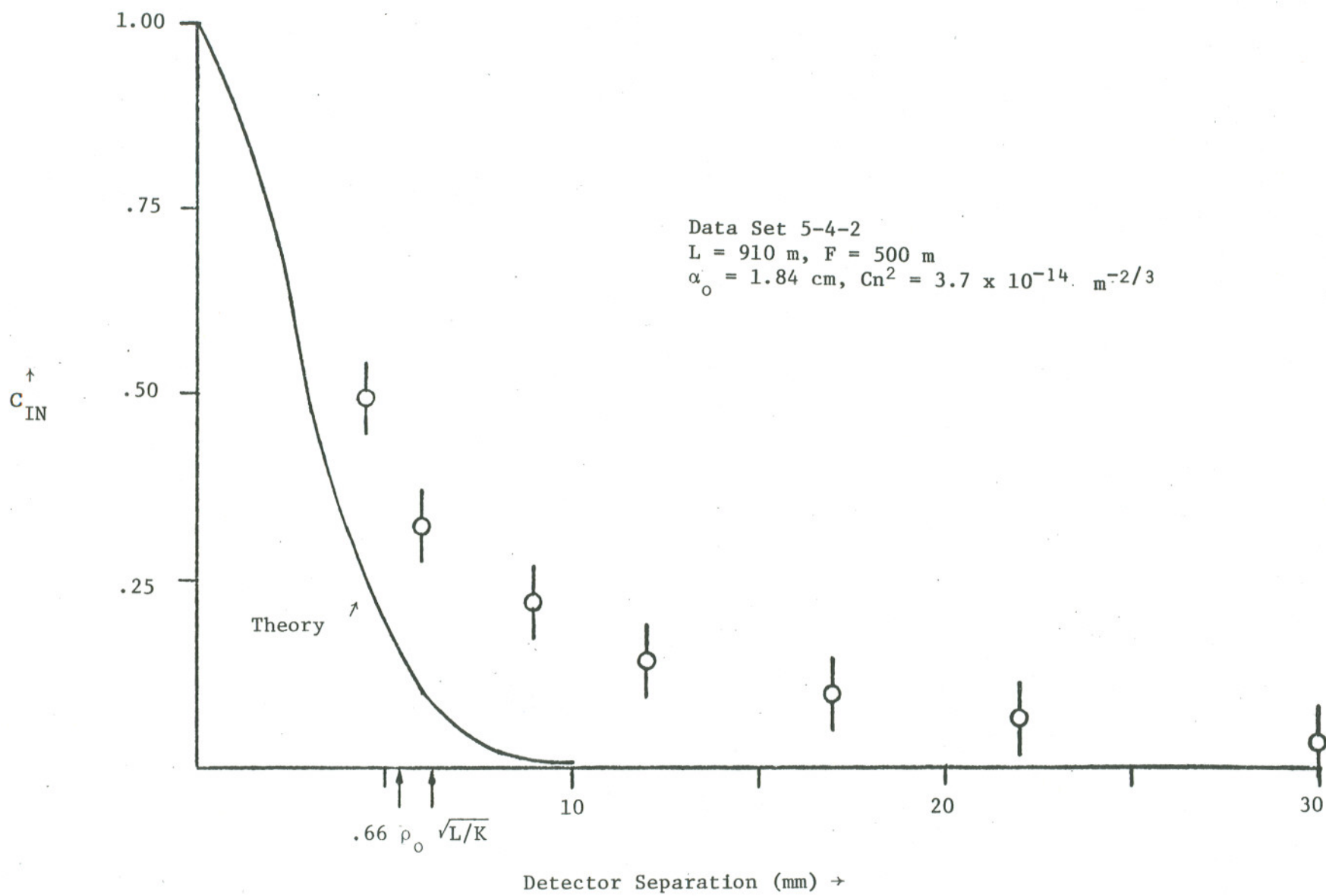


Figure 12 - Normalized Covariance Data

Figure 12 represents a data set taken at 910 meters while the focus was at 500 meters. The curve is nearly identical to that in Figure 8, for which the turbulence level is about the same. At this level of turbulence the outgoing beam is so distorted by the atmosphere that the focus information is destroyed.

In Figure 13, covariance scale size ($1/e^2$ point of covariance) is plotted against C_n^2 at both 910 and 500 meters. At higher turbulence levels where experiment deviates from theory, one can note the apparent lower limit of covariance length as being on the order of $\sqrt{L/K}$. This is the scale size normally associated with point source log normal statistics and the amplitude terms in the atmospheric transfer function.⁵

These pronounced deviations from the jointly gaussian theory indicate that the amplitude perturbation term in the general expression for the covariance function may be important in the realm where $\rho_0 < \sqrt{L/K}$.^{5,9,11} In fact it was these results that led to a reevaluation of the theory and subsequent recognition of the additive nature of the effect of the amplitude term which was lost in the phase only theory.

C. Variance Results

Figure 14 is a plot of normalized variance vs. the log amplitude standard deviation ($\sigma_\chi = .352k^{7/12}L^{11/12}$) which is a parameter relating to the effect of the amplitude perturbations in the atmosphere on the intensity variations of a spherical wave propagating in a homogeneous, locally isotropic, random medium.⁷

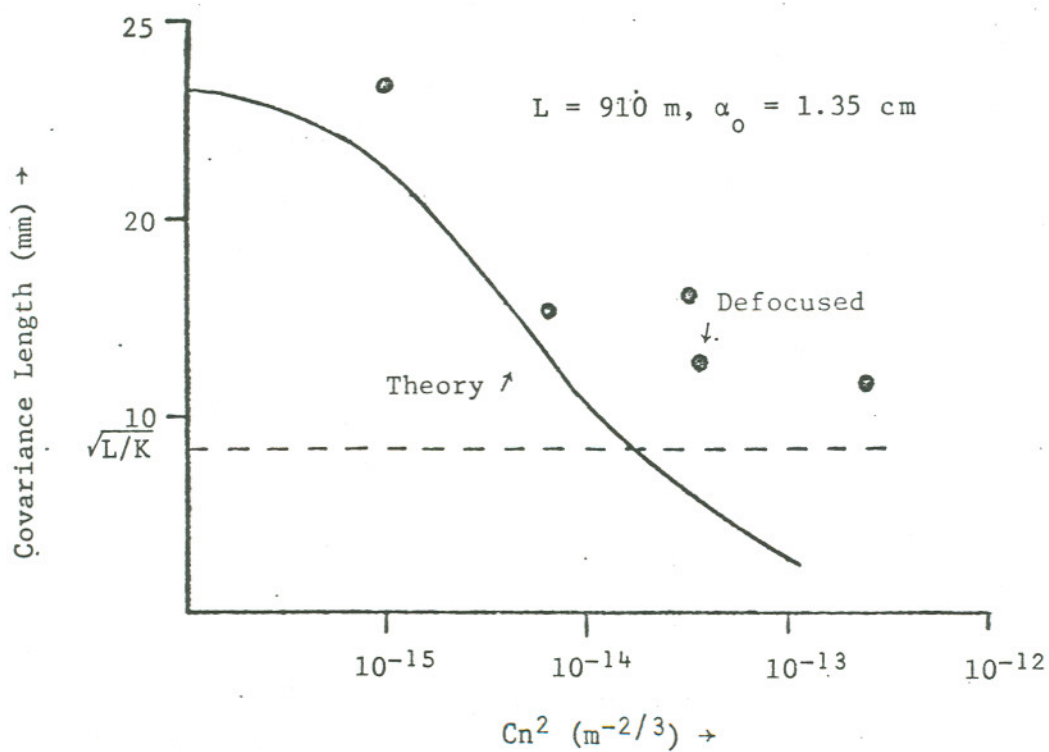
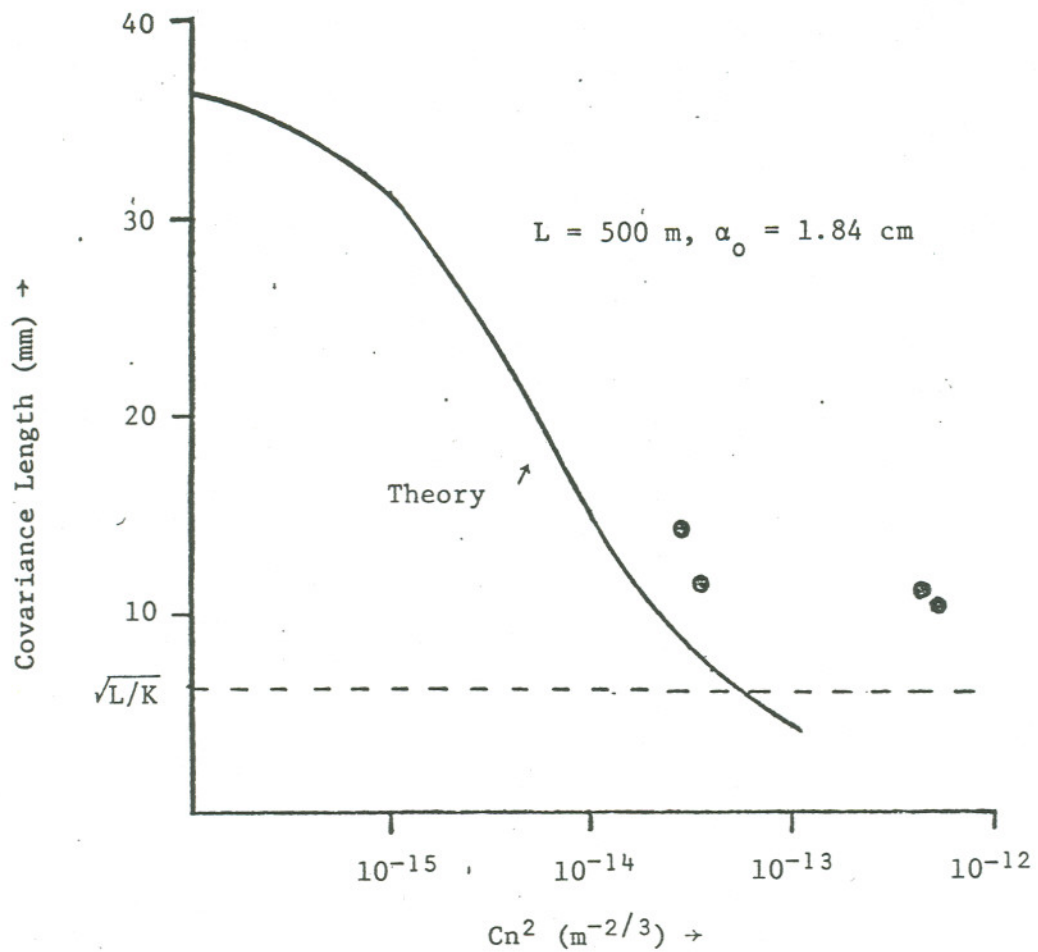


Figure 13 - Covariance Length vs Turbulence

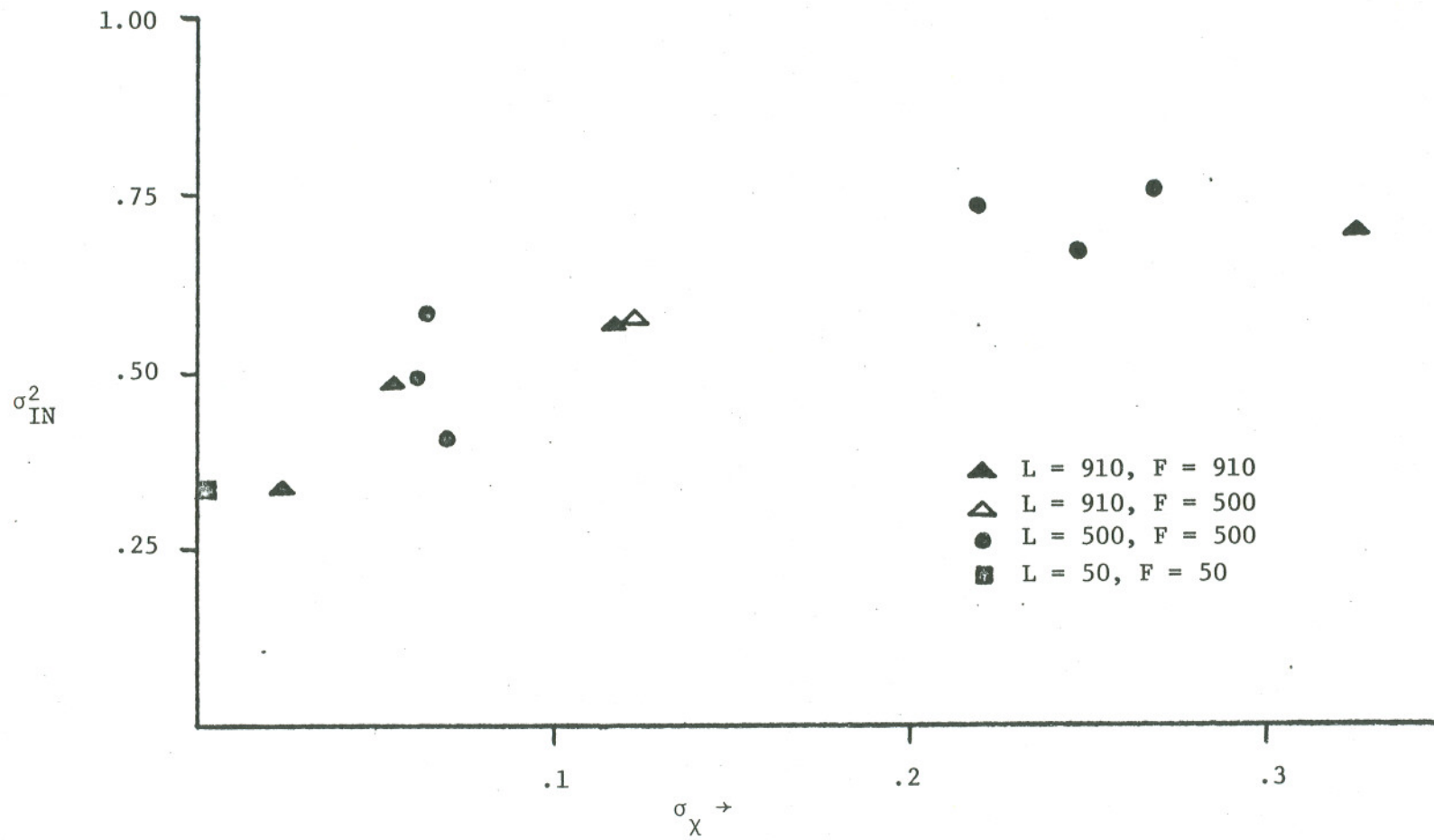


Figure 14 - Normalized Variance vs Log Amplitude Standard Deviation

The normalized variance taken at 50 meters ($\sigma_{IN}^2 = .33$) is entirely a source-target result with no atmospheric effects. Under the conditions at that time ($C_n^2 = 10^{-15}$) a stationary, nearly diffraction limited spot was observed on the target. This measurement does not agree with the earlier one made in the laboratory ($\sigma_{IN}^2 = 1$).

Both measurements were taken from the same target, the primary difference being the illuminating laser. The laboratory result was obtained using a 4 milliwatt TEM₀₀ Helium-Neon laser operating at 632.8 nm. The 50 meter measurement was made using the 1 watt Argon Ion laser operating at 488 nm.

Subsequent normalized variance measurements were made in a laboratory environment using a modified Coherent Radiation Model 52 Ion laser filled with a mixture of Argon and Krypton. A similar lack of contrast in the speckle pattern was observed.

Since this laser was tunable from blue to red (488 nm to 647.1 nm) measurements were made at various frequencies to rule out any frequency dependence due to the target material. Although some variation in contrast was observed with different wavelengths, there appeared to be no consistent frequency effect. The maximum normalized variance ($\sigma_{IN}^2 = .46$) was observed on the 647.1 nm line.

The difference between the results with the Argon laser and the unity normalized variance achieved with the He-Ne laser has been attributed to comparatively poor temporal coherence exhibited by the Argon laser. As pointed out by Bloom,¹² the ionized gas lasers exhibit

a considerably larger Doppler bandwidth than a neutral gas laser like He-Ne. The Argon laser used in these experiments has a long cavity therefore there are many longitudinal modes excited under the broad Doppler band.

Despite the failure of the source and target to produce the perfectly diffuse return that is implicit in the theoretical development, there are still some useful interpretations to be made about the normalized variance results.

$$\text{Let } \sigma_{\text{IN}}^2 = \sigma_{\text{P}}^2 + \sigma_{\text{A}}^2$$

σ_{P}^2 = variance due to target and atmospheric phase perturbations ($\sigma_{\text{P}}^2 = 1$ for perfectly diffuse target and coherent source).

σ_{A}^2 = variance due to atmospheric amplitude perturbations (see Ref. 5 for an integral expression).

In Figure 14, the $\sigma_{\text{IN}}^2 = .33$ can be taken as σ_{P}^2 , then the rise in variance above this point is due to the increase in σ_{A}^2 . Since the phase is completely scrambled by the target, then no amount of phase perturbation by the atmosphere will increase σ_{P}^2 .

Although these results do not allow exact comparison to the general theory, they are of practical interest as they stand, since many laser sources of interest are not perfectly coherent.

IV. CONCLUSION

As shown by the covariance data, the jointly gaussian theory appears to be valid only where $\rho_0 > \sqrt{L/K}$. In higher turbulence, the $\sqrt{L/K}$ scale size becomes dominant and may imply that the amplitude perturbation terms are important. It can be shown that the poor temporal coherence of the laser will not markedly affect the covariance results. Assume that there are two optical frequencies present in the transmitted light denoted by A and B.

$$C_I = \langle (I_1 - \bar{I}_1)(I_2 - \bar{I}_2) \rangle = \langle I_1 I_2 \rangle - \bar{I}_1 \bar{I}_2$$

$$I_1 = I_{1A} + I_{1B} \quad I_2 = I_{2A} + I_{2B}$$

$$C_I = \langle I_{1A} I_{2A} \rangle + \langle I_{1B} I_{2B} \rangle + \langle I_{1A} I_{2B} \rangle + \langle I_{1B} I_{2A} \rangle \\ - \bar{I}_{1A} \bar{I}_{2A} - \bar{I}_{1B} \bar{I}_{2B} - \bar{I}_{1A} \bar{I}_{2B} - \bar{I}_{1B} \bar{I}_{2A}$$

Since the difference frequency between two axial modes in the laser is far outside the detection bandwidth of our receivers, $\langle I_{1A} I_{2B} \rangle = \bar{I}_{1A} \bar{I}_{2B}$ and $\langle I_{1B} I_{2A} \rangle = \bar{I}_{1B} \bar{I}_{2A}$.

$$C_I = \langle I_{1A} I_{2A} \rangle - \bar{I}_{1A} \bar{I}_{2A} + \langle I_{1B} I_{2B} \rangle - \bar{I}_{1B} \bar{I}_{2B}$$

$$C_I = C_{IA} + C_{IB}$$

On an optical scale of frequency, A and B are very close, therefore

$$C_{IA} = C_{IB}$$

$$C_I = 2C_{IA}$$

$$C_{IN}(\rho) = \frac{C_I(\rho)}{C_I(o)} = \frac{2C_{IA}(\rho)}{2C_{IA}(o)} = C_{INA}(\rho)$$

This expression indicates that the normalized covariance function is the same whether one or two axial modes is present in the transmitted signal. This analysis can also be extended to several axial modes with the same result.

The variance results are not as definitive as those for the covariance, although as numerical solutions are obtained from the general theory and further experimental results are obtained, they may be useful for more detailed comparisons. When the amplitude term can be evaluated, then it can be separated and evaluated as a function of turbulence.

There remains a great deal of interesting work to be done regarding the propagation of speckle through the atmosphere. It is desirable to find a source-target combination that will provide target speckles of unity variance and adequate SNR so that comparisons with theory can be made more readily. Many more realms of relative size of the three basic scale size parameters ($\alpha_0, \rho_0, \sqrt{L/K}$) remain to be investigated. The relationship between the statistics and target detail remains to be studied. The spatial and temporal spectra of the intensity warrant investigation, in particular in how they relate to the remote wind sensing problem.

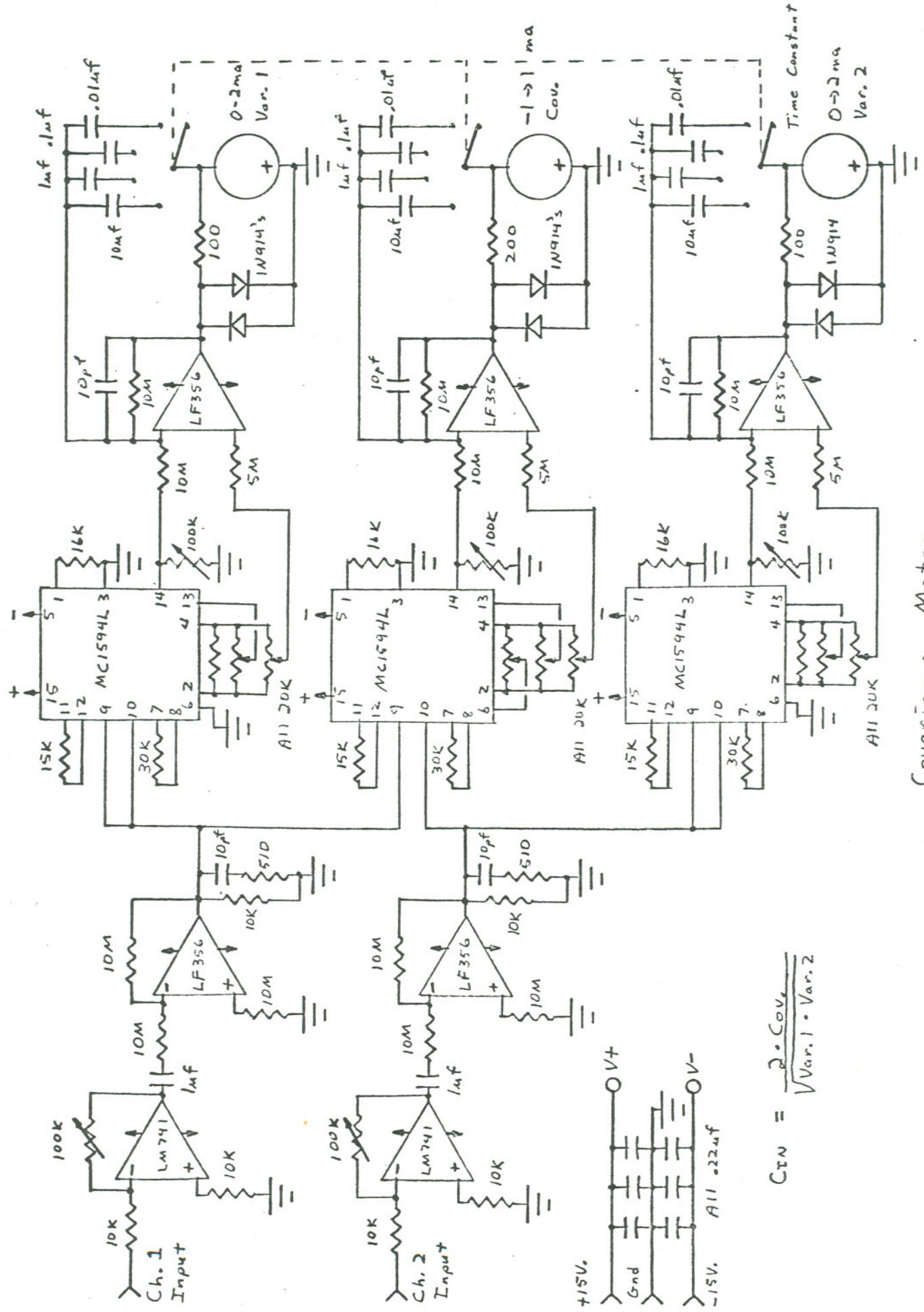
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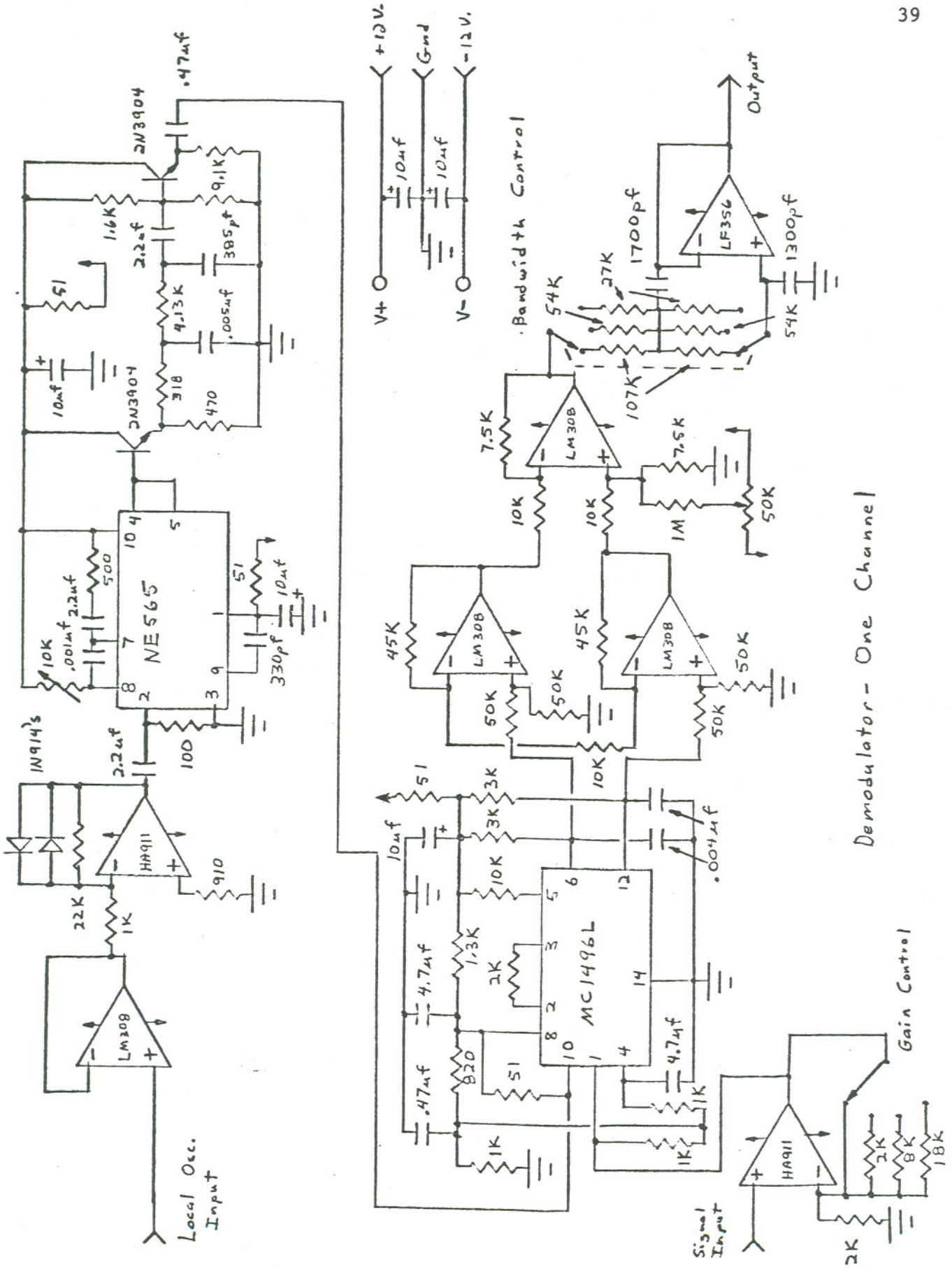
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APPENDIX

Circuit Schematics



Covariance Meter



Demodulator - One Channel